On the Efficiency of Calibrated Combined Ratio Estimators in Stratified Random Sampling

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***Abstract* — This study considered the modification of combined ratio estimators in stratified random sampling using calibration estimation approaches. The calibration distance measures with their associate constraints were used to modify the calibrated combine ratio estimators. New sets of calibration weights were derived and used to obtained new calibrated estimators of population mean. Empirical study through simulation was conducted to investigate the efficiency of the new estimators obtained. The results show that the proposed estimators are more efficient than the existing estimators considered in the study.**

Keywords: *Calibration estimation, Stratified sampling, Combined Ratio, Auxiliary variable.*

I. Introduction

Calibration estimation is the process of adjusting the original design weights to incorporate the known population totals of auxiliary variables. A calibration estimator uses calibrated weights that are determined to minimize a given distance measure to the original design weights while satisfying a set of constraints related to the auxiliary information. The calibration approach is used in stratified random sampling to obtain optimum strata weights for improving the precision of survey estimates of population parameters.

The technique of estimation by calibration in survey sampling was introduced by Deville and Sarndal(1992). The idea is to use auxiliary information to obtain a better estimate of a population statistic. Following Deville and Sarndal (1992), many researchers have studied calibration estimation by using different calibration constraints in survey sampling design. Singh *et al* (1998) are the first to extend the calibration approach to a stratified sampling design. Singh (2003), Tracy *et al* (2003), Kim *et al* (2007) and Clement and Enang (2015) applied calibration estimation to ratio-type estimators in stratified sampling. Rao *et al* (2012) proposed a multivariate calibration estimator for the population mean using different distance measures with two auxiliary variables in stratified sampling. Koyuncu and Kadilar (2016) have suggested calibration estimators for estimating the population mean in stratified sampling with using different calibration constraints based on auxiliary information.

II. Notations and Review of existing Estimators

Consider a finite population T of N elements, $\mathrm{T}=\{T_1,T_2,T_3,...,T_N\}$ consisting of L strata with N_h units in the hth stratum from which a simple random of size n_h is taken from the population using SRSWOR. Total Population size $N=\sum_{h=1}^{L}N_h$, sample size $n=\sum_{h=1}^{L}n_h$ where $y_{hi}, i=1,2,...,N_{hi}$ and $x_{hi}, i=1,2,...,N_{hi}$ of study variable y and x auxiliary variable. Let $W_h=N_h/N$ be the strata weights, $\bar{y}_h=n^{-1}\sum_{i=1}^{n_h}y_{hi}$ and $\bar{Y}_h=N^{-1}\sum_{i=1}^{n_h}y_{hi}$ are the sample and population means respectively for the study variables.
According to Cochran (1977), the traditional estimator of population mean in stratified sampling given as:

$$\bar{y}_{st}=\sum_{h=1}^{L}W_h\bar{y}_h \tag{2.0}$$

$$V\left(\bar{y}_{st}\right)=\sum_{h=1}^{L}W_h^2\left(\frac{1-f_h}{n_h}\right)s_{hy}^2 \qquad (2.1)$$

where $s_{hy}^2=\left(n_h-1\right)^{-1}\sum_{h=1}^{n_h}\left(y_{hi}-\bar{y}_h\right)^2$

Hansen *et al*. (1946) suggested a combined ratio estimator as

$$\bar{y}_{st}^{RC}=\frac{\sum_{i=1}^{n_h}W_h\bar{y}_h}{\sum_{i=1}^{n_h}W_h\bar{x}_h}\bar{X} \qquad (2.2)$$

The variance of the combined ratio estimator is

$$V\left(\bar{y}_{st}^{RC}\right)=\sum_{h=1}^{L}W_h^2\gamma_h\left(S_{yh}^2+R^2S_{xh}^2-2RS_{yxh}^2\right) \qquad (2.3)$$

where $R=\frac{\bar{Y}}{\bar{X}}$

Rao *et al.* (2016) proposed a new calibration scheme by incorporating coefficient of variation in the constraint to the chi-square distance function for the new calibration weight defined to improve the precision of the sample mean estimator in stratified random sampling, they considered coefficient of variation in place of variable in the work of Tracy *et al.* (2003) since it is a stable parameter and can resist the influence of outliers.

2.1 *Calibration Estimator I*

The scheme proposed is as follows:

$$\bar{y}_{st}^{R}=\sum_{h=1}^{L}\Omega_h^R\bar{y}_h \qquad (2.4)$$

where Ω_h^R is the calibrated weights such that the chi-square function Z is defined as

$$\left.\begin{aligned}&Min\,Z=\sum_{h=1}^{L}\left(\Omega_h^R-\Omega_h\right)^2/\Omega_hQ_h\\&s.t.\ \sum_{h=1}^{L}\Omega_h^R\left(\bar{x}_h+c_{xh}\right)=\sum_{h=1}^{L}\Omega_h\left(\bar{X}_h+C_{xh}\right)\end{aligned}\right\} \qquad (2.5)$$

where

$c_{xh}=s_{xh}/\bar{x}_h, C_{Xh}=S_{Xh}\big/\bar{X}_h, s_{xh}^2=(n_h-1)^{-1}\sum_{h=1}^{L}(x_{hi}-\bar{x}_h)^2, \bar{x}_h=n_h^{-1}\sum_{h=1}^{L}x_{hi}, S_{Xh}^2=(N_h-1)^{-1}\sum_{h=1}^{L}(x_{hi}-\bar{X}_h)^2$ The calibrated weights were obtained as

$$\Omega_h^R=\Omega_h+\frac{\Omega_hQ_h\left(\bar{x}_h+c_{xh}\right)}{\sum_{h=1}^{L}\Omega_hQ_h\left(\bar{x}_h+c_{xh}\right)^2}\left(\sum_{h=1}^{L}\Omega_h\left(\bar{X}_h+C_{xh}\right)-\sum_{h=1}^{L}\Omega_h\left(\bar{x}_h+c_{xh}\right)\right) \qquad (2.6)$$

and a new estimator was obtained as

$$\bar{y}_{st}^{R}=\sum_{h=1}^{L}\Omega_h\bar{y}_h+\hat{\beta}\left(\sum_{h=1}^{L}\Omega_h\left(\bar{X}_h+C_{xh}\right)-\sum_{h=1}^{L}\Omega_h\left(\bar{x}_h+c_{xh}\right)\right) \qquad (2.7)$$

where $\hat{\beta}=\frac{\sum_{h=1}^{L}\Omega_hQ_h\bar{y}_h\left(\bar{x}_h+c_{xh}\right)}{\sum_{h=1}^{L}\Omega_hQ_h\left(\bar{x}_h+c_{xh}\right)^2}$

$Q_h=(\bar{x}_h+c_x)^{-1}$

2.2 *Calibration Estimator II*

Rao *et al.* (2016) developed another calibration estimator as

$$\bar{y}_{st}^{\otimes}=\sum_{h=1}^{L}\Omega_h^{\otimes}\bar{y}_h \qquad (2.8)$$

where $\Omega_h^{\otimes}$ is the calibrated weights such that the chi-square function Z is defined as

$$\left.\begin{aligned} &Min\,Z^{\otimes}=\sum_{h=1}^{L}\left(\Omega_h^{\otimes}-\Omega_h\right)^2/\Omega_h Q_h \\ &s.t.\ \sum_{h=1}^{L}\Omega_h^{\otimes}\left(1+\bar{x}_h+c_{xh}\right)=\sum_{h=1}^{L}\Omega_h\left(1+\bar{X}_h+C_{xh}\right)\end{aligned}\right\} \quad (2.9)$$

The calibrated weights was obtained as

$$\Omega_h^{\otimes}=\Omega_h+\frac{\Omega_h Q_h\left(1+\bar{x}_h+c_{xh}\right)}{\sum_{h=1}^{L}\Omega_h Q_h\left(1+\bar{x}_h+c_{xh}\right)^2}\left(\sum_{h=1}^{L}\Omega_h\left(\bar{X}_h+C_{xh}\right)-\sum_{h=1}^{L}\Omega_h\left(\bar{x}_h+c_{xh}\right)\right) \quad (2.11)$$

and new estimator was obtained as

$$\bar{y}_{st}^{\otimes}=\sum_{h=1}^{L}\Omega_h\bar{y}_h+\hat{\beta}^{\otimes}\left(\sum_{h=1}^{L}\Omega_h\left(\bar{X}_h+C_{xh}\right)-\sum_{h=1}^{L}\Omega_h\left(\bar{x}_h+c_{xh}\right)\right) \quad (2.12)$$

where $\hat{\beta}^{\otimes}=\dfrac{\sum_{h=1}^{L}\Omega_h Q_h\bar{y}_h\left(1+\bar{x}_h+c_{xh}\right)}{\sum_{h=1}^{L}\Omega_h Q_h\left(1+\bar{x}_h+c_{xh}\right)^2}$

Having studied Rao *et al*. (2016) calibrated estimators. We combined ratio estimation concept to Rao *et al*. (2016) calibrated estimators in order to reduce the influence of outliers or extreme values on the estimators so as to increase the precision of the estimate of the population mean and give a highly efficient class of estimators of population mean.

III. Proposed Estimators

Conventional Combined Ratio Estimator in stratified random sampling given in (2.0) can be written as

$$\bar{y}_{st}^{RC}=\sum_{h=1}^{L}W_h^*\bar{y}_h \quad (3.0)$$

where $W_h^*=W_h\bar{X}\Big/\sum_{h=1}^{L}W_h\bar{x}_h$

3.1 *New Calibration Estimator I*

Motivated by Rao *et al.* (2016), calibrated combined ratio estimator denoted by $\bar{y}_{st}^{M}$ is proposed as:

$$\bar{y}_{st1}^{M}=\sum_{h=1}^{L}\Omega_h^M\bar{y}_h \quad (3.1)$$

where Ω_h^M is the new calibration weights minimizing the Chi-square distance measure (Z) subject to the calibration constraint and Q_h is suitably chosen weights which decide different forms of estimators given by

$$\left.\begin{aligned} &\min\ Z^*=\sum_{h=1}^{L}\left(\Omega_h^M-W_h^*\right)^2\Big/W_h^*Q_h \\ &s.t.\ \sum_{h=1}^{L}\Omega_h^M\left(\bar{x}_h+c_{xh}\right)=\sum_{h=1}^{L}W_h^*\left(\bar{X}_h+C_{xh}\right)\end{aligned}\right\} \quad (3.2)$$

To compute new calibrated weights $\left(\Omega_h^M\right)$, Lagrange multipliers technique is used and we have:

$$L_1=\sum_{h=1}^{L}\frac{\left(\Omega_h^M-W_h^*\right)^2}{W_h^*Q_h}-2\lambda_1\left(\sum_{h=1}^{L}\Omega_h^M\left(\bar{x}_h+c_{xh}\right)-\sum_{h=1}^{L}W_h^*\left(\bar{X}_h+C_{xh}\right)\right) \quad (3.3)$$

where λ_1 is the Lagrange's multiplier

Differentiate (3.3) partially with respect to Ω_h^M, and λ_1, and equal to zero, give (3.4) and (3.5) respectively

$$\Omega_h^M = W_h^* + \lambda_1 W_h^* Q_h \left(\bar{x}_h + c_{xh}\right) \tag{3.4}$$

$$\sum_{h=1}^{L} \Omega_h^M \left(\bar{x}_h + c_{xh}\right) - \sum_{h=1}^{L} W_h^* \left(\bar{X}_h + C_{xh}\right) = 0 \tag{3.5}$$

Substitute (3.4) in (3.5), the result is obtained as:

$$\lambda_1 \sum_{h=1}^{L} W_h^* Q_h \left(\bar{x}_h + c_{xh}\right)^2 = \sum_{h=1}^{L} W_h^* \left(\bar{X}_h + C_{xh}\right) - \sum_{h=1}^{L} W_h^* \left(\bar{x}_h + c_{xh}\right) \tag{3.6}$$

$$\lambda_1 = \frac{\sum_{h=1}^{L} W_h^* \left(\bar{X}_h + C_{xh}\right) - \sum_{h=1}^{L} W_h^* \left(\bar{x}_h + c_{xh}\right)}{\sum_{h=1}^{L} W_h^* Q_h \left(\bar{x}_h + c_{xh}\right)^2} \tag{3.7}$$

On substituting (3.7) in (3.4), the calibrated weights Ω_h^M can be written as:

$$\Omega_h^M = W_h^* + W_h^* Q_h \left(\bar{x}_h + c_{xh}\right) \left(\frac{\sum_{h=1}^{L} W_h^* \left(\bar{X}_h + C_{xh}\right) - \sum_{h=1}^{L} W_h^* \left(\bar{x}_h + c_{xh}\right)}{\sum_{h=1}^{L} W_h^* Q_h \left(\bar{x}_h + c_{xh}\right)^2} \right) \tag{3.8}$$

Substituting (3.8) in (3.0), obtain the new combined calibration estimator $\left(\bar{y}_{st}^{M}\right)$ as:

$$\bar{y}_{st1}^M = \sum_{h=1}^{L} W_h^* \bar{y}_h + \hat{\beta}_1 \sum_{h=1}^{L} W_h^* \left(\left(\bar{X}_h + C_{xh}\right) - \left(\bar{x}_h + c_{xh}\right) \right) \tag{3.9}$$

Substituting $W_h^* = W_h \bar{X} / \sum_{h=1}^{L} W_h \bar{x}_h$ in (3.9), gives

$$\bar{y}_{st1}^M = \bar{X} \frac{\sum_{h=1}^{L} W_h \bar{y}_h}{\sum_{h=1}^{L} W_h \bar{x}_h} + \hat{\beta}_1 \frac{\bar{X} \sum_{h=1}^{L} W_h}{\sum_{h=1}^{L} W_h \bar{x}_h} \left(\left(\bar{X}_h + C_{xh}\right) - \left(\bar{x}_h + c_{xh}\right) \right) \tag{3.11}$$

where $\hat{\beta}_1 = \frac{\sum_{h=1}^{L} W_h Q_h \bar{y}_h \left(\bar{x}_h + c_{xh}\right)}{\sum_{h=1}^{L} W_h Q_h \left(\bar{x}_h + c_{xh}\right)^2}$

By setting $Q_h = 1$, $Q_h = 1/\bar{x}_h$ and $Q_h = \left(\bar{x}_h + c_{xh}\right)^{-1}$ set of new estimators are obtained respectively as:

$$\bar{y}_{st11}^M = \bar{X} \frac{\sum_{h=1}^{L} W_h \bar{y}_h}{\sum_{h=1}^{L} W_h \bar{x}_h} + \hat{\beta}_{11} \frac{\bar{X} \sum_{h=1}^{L} W_h}{\sum_{h=1}^{L} W_h \bar{x}_h} \left(\left(\bar{X}_h + C_{xh}\right) - \left(\bar{x}_h + c_{xh}\right) \right) \tag{3.12}$$

$$\bar{y}_{st12}^{M} = \bar{X}\frac{\sum_{h=1}^{L} W_h \bar{y}_h}{\sum_{h=1}^{L} W_h \bar{x}_h} + \hat{\beta}_{12}\frac{\bar{X}\sum_{h=1}^{L} W_h}{\sum_{h=1}^{L} W_h \bar{x}_h}\left(\left(\bar{X}_h + C_{xh}\right) - \left(\bar{x}_h + c_{xh}\right)\right) \quad (3.13)$$

$$\bar{y}_{st13}^{M} = \bar{X}\frac{\sum_{h=1}^{L} W_h \bar{y}_h}{\sum_{h=1}^{L} W_h \bar{x}_h} + \hat{\beta}_{13}\frac{\bar{X}\sum_{h=1}^{L} W_h}{\sum_{h=1}^{L} W_h \bar{x}_h}\left(\left(\bar{X}_h + C_{xh}\right) - \left(\bar{x}_h + c_{xh}\right)\right) \quad (3.14)$$

where $\hat{\beta}_{11} = \frac{\sum_{h=1}^{L} W_h \bar{y}_h \left(\bar{x}_h + c_{xh}\right)}{\sum_{h=1}^{L} W_h \left(\bar{x}_h + c_{xh}\right)^2}$, $\hat{\beta}_{12} = \frac{\sum_{h=1}^{L} W_h \bar{x}_h^{-1} \bar{y}_h \left(\bar{x}_h + c_{xh}\right)}{\sum_{h=1}^{L} W_h \bar{x}_h^{-1} \left(\bar{x}_h + c_{xh}\right)^2}$, $\hat{\beta}_{13} = \frac{\sum_{h=1}^{L} W_h \bar{y}_h}{\sum_{h=1}^{L} W_h \left(\bar{x}_h + c_{xh}\right)}$

3.2 *New Calibration Estimator II*

$$\bar{y}_{st2}^{M} = \sum_{h=1}^{L} \Omega_h^{M\bullet} \bar{y}_h \quad (3.15)$$

$$\left.\begin{aligned} \min\ Z_2 &= \sum_{h=1}^{L}\left(\Omega_h^{M\bullet} - W_h^*\right)^2 \Big/ W_h^* Q_h \\ s.t.\ \sum_{h=1}^{L} \Omega_h^{M\bullet}\left(1 + \bar{x}_h + c_{xh}\right) &= \sum_{h=1}^{L} W_h^*\left(1 + \bar{X}_h + C_{xh}\right) \end{aligned}\right\} \quad (3.16)$$

To compute new calibrated weights $\left(\Omega_h^{M\bullet}\right)$, Lagrange multipliers technique is used and we have:

$$L_2 = \sum_{h=1}^{L}\frac{\left(\Omega_h^{M\bullet} - W_h^*\right)^2}{W_h^* Q_h} - 2\lambda_2\left(\sum_{h=1}^{L}\Omega_h^{M\bullet}\left(1 + \bar{x}_h + c_{xh}\right) - \sum_{h=1}^{L} W_h^*\left(1 + \bar{X}_h + C_{xh}\right)\right) \quad (3.17)$$

Differentiate (3.17) partially with respect to $\Omega_h^{M\bullet}$, and λ_2, equal to zero

$$\Omega_h^{M\bullet} = W_h^* + \lambda_2 W_h^* Q_h\left(1 + \bar{x}_h + c_{xh}\right) \quad (3.18)$$

$$\sum_{h=1}^{L}\Omega_h^{M\bullet}\left(1 + \bar{x}_h + c_{xh}\right) - \sum_{h=1}^{L} W_h^*\left(1 + \bar{X}_h + C_{xh}\right) = 0 \quad (3.19)$$

Substitute (3.18) in (3.19), the result is obtained as:

$$\lambda_2\sum_{h=1}^{L} W_h^* Q_h\left(1 + \bar{x}_h + c_{xh}\right)^2 = \sum_{h=1}^{L} W_h^*\left(1 + \bar{X}_h + C_{xh}\right) - \sum_{h=1}^{L} W_h^*\left(1 + \bar{x}_h + c_{xh}\right) \quad (3.20)$$

$$\lambda_2 = \frac{\sum_{h=1}^{L} W_h^*\left(1 + \bar{X}_h + C_{xh}\right) - \sum_{h=1}^{L} W_h^*\left(1 + \bar{x}_h + c_{xh}\right)}{\sum_{h=1}^{L} W_h^* Q_h\left(1 + \bar{x}_h + c_{xh}\right)^2} \quad (3.21)$$

Substitute λ_2 in (3.21) and then put the result in (3.18), obtained (3.22)

$$\Omega_h^{M\bullet} = W_h^* + W_h^* Q_h\left(1+\bar{x}_h + c_{xh}\right)\left(\frac{\sum_{h=1}^{L} W_h^*\left(1+\bar{X}_h + C_{xh}\right) - \sum_{h=1}^{L} W_h^*\left(1+\bar{x}_h + c_{xh}\right)}{\sum_{h=1}^{L} W_h^* Q_h\left(1+\bar{x}_h + c_{xh}\right)^2}\right) \quad (3.22)$$

$$\bar{y}_{st2}^{M} = \sum_{h=1}^{L} W_h^* \bar{y}_h + \lambda_2 \sum_{h=1}^{L} W_h^* Q_h\left(1+\bar{x}_h + c_{xh}\right)\bar{y}_h \quad (3.23)$$

$$\bar{y}_{st2}^{M} = \sum_{h=1}^{L} W_h^* \bar{y}_h + \hat{\beta}_{(M)1} \sum_{h=1}^{L} W_h^* \left(\left(1+\bar{X}_h + C_{xh}\right) - \left(1+\bar{x}_h + c_{xh}\right)\right) \quad (3.24)$$

where $\hat{\beta}_{(M)1} = \dfrac{\sum_{h=1}^{L} W_h Q_h \bar{y}_h\left(1+\bar{x}_h + c_{xh}\right)}{\sum_{h=1}^{L} W_h Q_h\left(1+\bar{x}_h + c_{xh}\right)^2}$

By setting $Q_h = 1$, $Q_h = 1/\bar{x}_h$ and $Q_h = \left(1+\bar{x}_h + c_{xh}\right)^{-1}$ set of new estimators were obtained respectively as:

$$\bar{y}_{st21}^{M} = \bar{X}\frac{\sum_{h=1}^{L} W_h \bar{y}_h}{\sum_{h=1}^{L} W_h \bar{x}_h} + \hat{\beta}_{(M)21}\frac{\bar{X}\sum_{h=1}^{L} W_h}{\sum_{h=1}^{L} W_h \bar{x}_h}\left(\left(1+\bar{X}_h + C_{xh}\right) - \left(1+\bar{x}_h + c_{xh}\right)\right) \quad (3.25)$$

$$\bar{y}_{st22}^{M} = \bar{X}\frac{\sum_{h=1}^{L} W_h \bar{y}_h}{\sum_{h=1}^{L} W_h \bar{x}_h} + \hat{\beta}_{(M)22}\frac{\bar{X}\sum_{h=1}^{L} W_h}{\sum_{h=1}^{L} W_h \bar{x}_h}\left(\left(1+\bar{X}_h + C_{xh}\right) - \left(1+\bar{x}_h + c_{xh}\right)\right) \quad (3.26)$$

$$\bar{y}_{st23}^{M} = \bar{X}\frac{\sum_{h=1}^{L} W_h \bar{y}_h}{\sum_{h=1}^{L} W_h \bar{x}_h} + \hat{\beta}_{(M)23}\frac{\bar{X}\sum_{h=1}^{L} W_h}{\sum_{h=1}^{L} W_h \bar{x}_h}\left(\left(1+\bar{X}_h + C_{xh}\right) - \left(1+\bar{x}_h + c_{xh}\right)\right) \quad (3.27)$$

where $\hat{\beta}_{(M)21} = \dfrac{\sum_{h=1}^{L} W_h \bar{y}_h\left(1+\bar{x}_h + c_{xh}\right)}{\sum_{h=1}^{L} W_h\left(1+\bar{x}_h + c_{xh}\right)^2}$, $\hat{\beta}_{(M)22} = \dfrac{\sum_{h=1}^{L} W_h \bar{x}_h^{-1} \bar{y}_h\left(1+\bar{x}_h + c_{xh}\right)}{\sum_{h=1}^{L} W_h \bar{x}_h^{-1}\left(1+\bar{x}_h + c_{xh}\right)^2}$, and $\hat{\beta}_{(M)23} = \dfrac{\sum_{h=1}^{L} W_h \bar{y}_h}{\sum_{h=1}^{L} W_h\left(1+\bar{x}_h + c_{xh}\right)}$

III. Summary of Srudy

In this section, a simulation study was conducted to examine the superiority of the proposed estimators over other estimators considered in the study.

Data of size 1000 units were generated for study Populations stratified into 3 non-overlapping heterogeneous groups of 200, 300, and 500 using the function defined in Table 1. Samples of sizes 20, 30, and 50 were selected 10,000 times by method SRSWOR from each stratum respectively. The precision (PRE) of the considered estimators were computed using

$$MSE\left(\bar{y}_{st}\right) = \frac{1}{10000}\sum_{j=1}^{10000}\left(\bar{y}_{st} - \bar{Y}\right)^2 \quad (4.0)$$

$$MSE(\theta_i)=\frac{1}{10000}\sum_{j=1}^{10000}(\theta_i-\bar{Y})^2,\ \theta_i=\bar{y}_{st}^{RC},\ \bar{y}_{st}^{R},\bar{y}_{st}^{\otimes},\bar{y}_{st1}^{M},\bar{y}_{st2}^{M} \quad (4.1)$$

$$PRE(\theta_i)=\frac{MSE(\bar{y}_{st})}{MSE(\theta_i)}X100,\theta_i=\bar{y}_{st}^{RC},\ \bar{y}_{st}^{R},\bar{y}_{st}^{\otimes},\bar{y}_{st1}^{M},\bar{y}_{st2}^{M} \quad (4.2)$$

Table1: Populations Used for Empirical Study

Populations	*Auxiliary variable* X	*Study variable* y
I	$x_h \approx \exp(\theta_h),\theta_1=5,\theta_2=6,$ $\theta_3=4,h=1,2,3$	$y_{hi}=\alpha_h x_{hi}^2+\xi_{hi},\ \alpha_{1h}=E(x_h),$ $\alpha=0.5,\ \xi_h\approx N(0,1),h=1,2,3$
II	$x_h \approx gamma(\theta_h,\eta_h),\theta_1=3,\eta_1=2,$ $\theta_2=3,\eta_2=1,\theta_3=3,\eta_3=3,$	
III	$x_h \approx \text{chisq}(\theta_h),\theta_1=5,\theta_2=6,$ $\theta_3=4,h=1,2,3$	

Table 2: PREs of Some Existing and Proposed Estimator Using Population I

Estimator	$Model: y_{hi}=\alpha_h x_{hi}^2+\xi_{hi}$		
	$Q_h=1$	$Q_h=\bar{x}_h^{-1}$	$Q_h=(\bar{x}_h+c_{xh})^{-1}$
$\bar{y}_{st}$	100	100	100
Combined ratio $\bar{y}_{st}^{RC}$	232.5467	232.5467	232.5467
Rao *et al.* (2016) $\bar{y}_{st}^{R}$	197.0793	190.3512	192.3986
Proposed Estimator $\bar{y}_{st}^{M}$ Estimator	**505.5474**	**491.1103**	**495.75**
	$Model: y_{hi}=\alpha_h x_{hi}^3+\xi_{hi}$		
$\bar{y}_{st}$	100	100	100
Combined ratio $\bar{y}_{st}^{RC}$	154.11	154.11	154.11
Rao *et al.* (2016) $\bar{y}_{st}^{R}$	150.5576	145.7549	147.0729
Proposed Estimator $\bar{y}_{st}^{M}$ Estimator	**228.5307**	**222.2101**	**223.9917**
	$Model: y_{hi}=\alpha_h x_{hi}^4+\xi_{hi}$		
$\bar{y}_{st}$	100	100	100
Combined ratio $\bar{y}_{st}^{RC}$	127.7651	127.7651	127.7651
Rao *et al.* (2016) $\bar{y}_{st}^{R}$	128.8235	125.615	126.473
Proposed Estimator $\bar{y}_{st(M)}^{RC}$	**158.2605**	**155.1596**	**156.0107**

Table 3: PREs of Some Existing and Proposed Estimator Using Population II

Estimator	$Model: y_{hi} = \alpha_h x_{hi}^2 + \xi_{hi}$		
	$Q_h = 1$	$Q_h = \bar{x}_h^{-1}$	$Q_h = (\bar{x}_h + c_{xh})^{-1}$
$\bar{y}_{st}$	100	100	100
Combined ratio $\bar{y}_{st}^{RC}$	155.1702	155.1702	155.1702
Rao *et al.* (2016) $\bar{y}_{st}^{R}$	154.0309	140.6206	144.0594
Proposed Estimator $\bar{y}_{st}^{M}$	**214.6269**	**202.718**	**206.0825**
Estimator	$Model: y_{hi} = \alpha_h x_{hi}^3 + \xi_{hi}$		
$\bar{y}_{st}$	100	100	100
Combined ratio $\bar{y}_{st}^{RC}$	141.7887	141.7887	141.7887
Rao *et al.* (2016) $\bar{y}_{st}^{R}$	147.6528	131.4799	135.3618
Proposed Estimator $\bar{y}_{st}^{M}$	**207.7132**	**186.5198**	**191.7639**
Estimator	$Model: y_{hi} = \alpha_h x_{hi}^4 + \xi_{hi}$		
$\bar{y}_{st}$	100	100	100
Combined ratio $\bar{y}_{st}^{RC}$	123.2122	123.2122	123.2122
Rao *et al.* (2016) $\bar{y}_{st}^{R}$	127.9241	118.0011	120.3653
Proposed Estimator $\bar{y}_{st}^{M}$	**153.9231**	**143.4175**	**145.98**

Table 4: PREs of Some Existing and Proposed Estimator Using Population III

Estimator	$Model: y_{hi} = \alpha_h x_{hi}^2 + \xi_{hi}$		
	$Q_h = 1$	$Q_h = \bar{x}_h^{-1}$	$Q_h = \left(\bar{x}_h + c_{xh}\right)^{-1}$
$\bar{y}_{st}$	100	100	100
Combined ratio $\bar{y}_{st}^{RC}$	238.6025	238.6025	238.6025
Rao *et al.* (2016) $\bar{y}_{st}^{R}$	204.1169	184.7299	196.1744
Proposed Estimator $\bar{y}_{st}^{M}$	**505.7901**	**469.3246**	**492.6216**
Estimator	$Model: y_{hi} = \alpha_h x_{hi}^3 + \xi_{hi}$		
$\bar{y}_{st}$	100	100	100
Combined ratio $\bar{y}_{st}^{RC}$	161.049	161.049	161.049
Rao *et al.* (2016) $\bar{y}_{st}^{R}$	156.6157	145.7492	151.8573
Proposed Estimator $\bar{y}_{st}^{M}$	**250.5381**	**234.5052**	**243.7685**
Estimator	$Model: y_{hi} = \alpha_h x_{hi}^4 + \xi_{hi}$		
$\bar{y}_{st}$	100	100	100
Combined ratio $\bar{y}_{st}^{RC}$	130.3801	130.3801	130.3801
Rao *et al.* (2016) $\bar{y}_{st}^{R}$	130.9585	124.9782	128.2501
Proposal Estimator $\bar{y}_{st}^{M}$	**164.5409**	**158.3182**	**161.8038**

Table 5: PREs of Some Existing and Proposed Estimator Using Population I

Estimator	$Model: y_{hi} = \alpha_h x_{hi}^2 + \xi_{hi}$		
	$Q_h = 1$	$Q_h = \bar{x}_h^{-1}$	$Q_h = (1 + \bar{x}_h + c_{xh})^{-1}$
$\bar{y}_{st}$	100	100	100
Combined ratio $\bar{y}_{st}^{RC}$	232.5467	232.5467	232.5467
Rao *et al.* (2016) $\bar{y}_{st}^{\otimes}$	164.6595	159.8159	162.1221
Proposed Estimator $\bar{y}_{st2}^{M}$	**421.7624**	**409.3083**	**415.3679**
Estimator	$Model: y_{hi} = \alpha_h x_{hi}^3 + \xi_{hi}$		
$\bar{y}_{st}$	100	100	100
Combined ratio $\bar{y}_{st}^{RC}$	154.11	154.11	154.11
Rao *et al.* (2016) $\bar{y}_{st}^{\otimes}$	135.8736	132.1821	133.8524
Proposed Estimator $\bar{y}_{st2}^{M}$	**207.6716**	**202.593**	**204.9237**
Estimator	$Model: y_{hi} = \alpha_h x_{hi}^4 + \xi_{hi}$		
$\bar{y}_{st}$	100	100	100
Combined ratio $\bar{y}_{st}^{RC}$	127.7651	127.7651	127.7651
Rao *et al.* (2016) $\bar{y}_{st}^{\otimes}$	121.2595	118.6594	119.8126
Proposed Estimator $\bar{y}_{st2}^{M}$	**150.5604**	**147.9623**	**149.1289**

Table 6: PREs of Some Existing and Proposed Estimator Using Population II

Estimator	$Model: y_{hi} = \alpha_h x_{hi}^2 + \xi_{hi}$		
	$Q_h = 1$	$Q_h = \bar{x}_h^{-1}$	$Q_h = (1 + \bar{x}_h + c_{xh})^{-1}$
$\bar{y}_{st}$	100	100	100
Combined ratio $\bar{y}_{st}^{RC}$	155.1702	155.1702	155.1702
Rao *et al.* (2016) $\bar{y}_{st}^{\otimes}$	135.1992	124.6861	129.4649
Proposed Estimator $\bar{y}_{st2}^{M}$	**198.4107**	**186.6355**	**192.2473**
Estimator	$Model: y_{hi} = \alpha_h x_{hi}^3 + \xi_{hi}$		
$\bar{y}_{st}$	100	100	100
Combined ratio $\bar{y}_{st}^{RC}$	141.7887	141.7887	141.7887
Rao *et al.* (2016) $\bar{y}_{st}^{\otimes}$	130.0832	118.3879	123.4943
Proposed Estimator $\bar{y}_{st2}^{M}$	**184.1472**	**168.2123**	**175.2861**
Estimator	$Model: y_{hi} = \alpha_h x_{hi}^4 + \xi_{hi}$		
$\bar{y}_{st}$	100	100	100
Combined ratio $\bar{y}_{st}^{RC}$	123.2122	123.2122	123.2122
Rao *et al.* (2016) $\bar{y}_{st}^{\otimes}$	118.3073	110.7084	114.0037
Proposed Estimator $\bar{y}_{st2}^{M}$	**143.6076**	**135.35**	**138.9795**

Table 7: PREs of Some Existing and Proposed Estimator Using Population III

Estimator	$Model: y_{hi} = \alpha_h x_{hi}^2 + \xi_{hi}$		
	$Q_h = 1$	$Q_h = \bar{x}_h^{-1}$	$Q_h = (1+\bar{x}_h + c_{xh})^{-1}$
$\bar{y}_{st}$	100	100	100
Combined ratio $\bar{y}_{st}^{RC}$	238.6025	238.6025	238.6025
Rao *et al.* (2016) $\bar{y}_{st}^{\otimes}$	169.4123	155.225	165.0522
Proposed Estimator $\bar{y}_{st2}^{M}$ Estimator	**433.1277**	**397.9786**	**423.1307**
	$Model: y_{hi} = \alpha_h x_{hi}^3 + \xi_{hi}$		
$\bar{y}_{st}$	100	100	100
Combined ratio $\bar{y}_{st}^{RC}$	161.049	161.049	161.049
Rao *et al.* (2016) $\bar{y}_{st}^{\otimes}$	140.0151	131.402	137.1992
Proposed Estimator $\bar{y}_{st2}^{M}$ Estimator	**225.4846**	**212.1595**	**221.2694**
	$Model: y_{hi} = \alpha_h x_{hi}^4 + \xi_{hi}$		
$\bar{y}_{st}$	100	100	100
Combined ratio $\bar{y}_{st}^{RC}$	130.3801	130.3801	130.3801
Rao *et al.* (2016) $\bar{y}_{st}^{\otimes}$	122.8585	117.7912	121.1398
Proposed Estimator $\bar{y}_{st2}^{M}$	**155.9837**	**150.5291**	**154.1842**

4.1 Discussion of Results

Tables 2 – 7 show the Percentage Relative Efficiency (PRE) of the proposed combined calibration estimators and some existing estimators. The results revealed that all the proposed combined calibrated estimators have higher PRE compared to the conventional estimators and other existing estimators considered under stratified random sampling.

V. Conclusion

From the results obtained so far, the empirical study revealed the efficiency of the proposed calibration estimators over existing related estimators considered in the study, the proposed combined calibration estimators having higher percentage relative efficiency compared to some existing calibration estimators in the numerical analysis. This implies that the proposed combined calibration estimators are more efficient and can produce better estimates of the population mean than the stratified sample mean $(\bar{y}_{st})$ and other existing estimators considered in the study.

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