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Original Research Article

Regime and Order Selection for SETAR Nonlinear Time Series Model

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Abstract

The common empirical time series modeling assumes linearity and stationarity in the relationship between the variables. However, most applied time series research finds it difficult to assume linearity in data. Therefore, nonlinear models could be more representative of such data generation processes. To achieve this type of phenomenon in time series data, a suitable order and threshold regime number needs to be specified for nonlinear time series models which is the focus of this study. The nonlinear model considered in this study is the Self-Exiting Threshold Autoregressive (SETAR) model. The model is used to fit and forecast simulated nonlinear autoregressive functions at different sample sizes and steps ahead respectively. The SETAR of 2 and 3 autoregressive orders (p) within a regime and 2 and 3 regimes orders (d) are fitted at different sample sizes. The relative performances of the models [SETAR (p, d)] are examined to identify the best autoregressive and regime orders within the context of stationarity. Results showed that the SETAR (3, 2) and SETAR (2, 2) are the best for fitting small & moderate, and large sample sizes respectively in both the simulated and real-life data. Also, the best forecast models are SETAR (3, 2) followed by SETAR (2, 2) at different steps ahead. Finally, it is revealed that; there is an increase in fitting and forecasting performances of all the models when the sample sizes and the number of steps ahead are increased.

Keywords: SETAR Model, Regime order, Autoregressive order, simulation technique.

1.0 Introduction

Time series is a sequence of data collected sequentially over time. It involves consecutive observations on finite variables that are made over time (Cochrane, 2005). Typically, the observed data are sequential at fixed intervals (daily, monthly, quarterly, and yearly). Monthly data for unemployment, weekly supply and demand, quarterly interest rate and daily profit, day-to-day sales, yearly share prices, weekly flooding observed are time-series data.

There are two main purposes of time series analysis; these are to provide a model to a successive observation or understand the stochastic mechanism of a series of observation and to predict for upcoming observations from the present and past of that series and probably, additional associated factors and/or information (Akeyede et al, 2015).

The common empirical time series modeling assumes linearity and stationarity in the variables' relationship. However, most applied research finds it appropriate to assume linearity. Recently, arguments have been presented, that nonlinear specification may be a more realistic representation of data generation

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processes (Franses and van Dijk, 2000). To accommodate this kind of dynamic behavior using time series data, regime-switching models like threshold models have been introduced (Granger and Terasvirta, 1993). This study therefore aimed at determining suitable autoregressive and regime orders for the Self-Exiting Threshold Autoregressive (SETAR) model in fitting and forecasting nonlinear time series models under the assumption of stationarity.

1.1 *Nonlinear Autoregressive Models*

A nonlinear model is the one in which the relationship between two variables has a more broad functional dependency over time than the linear system of the model. Nonlinear models for time series are capable of capturing asymmetry, jump, wave, and other nonlinear behaviours, for instance, the inflation rates increase more rapidly than it falls. As far back as 1970, many classes of nonlinear time series modeling have been proposed.

It is well known that the nonlinear models offer a considerably broader range of likely dynamics in time series data than the linear counterpart. (Tong, 1978 and 1983] and (Tong and Lim, 1980) proposed the nonlinear autoregressive model which has a regime-switching technique explaining intermittent features of time series data and widely described in (Tong, 1990). This time series model that captures the vigorous characteristics of time series data by substituting the regimes is referred to as the Threshold Autoregressive (TAR) model. The structures of this form of time series models are jump marvels, wave, and limit cycles that cannot be captured by simple linear autoregressive function.

The application of the TAR model is commonly found in economic and financial time series most especially there are many works on price transmissions and agricultural marketing which shows that there is an irregularity in the change of prices at different levels of the marketing systems.

Currently, the TAR model bids us exciting likelihoods. The general form of the model proposed by Tong (1990) is presented as follows:

$$\mathrm{y_t} = \emptyset_0^{(s_t)} + \sum_{\mathrm{i=1}}^{\mathrm{p}} \emptyset_{\mathrm{i}}^{(s_t)} \mathrm{y_{t-1}} + e_t^{(s_t)}, \tag{1}$$

where $e_t^{(s_t))}$are independently and identically distributed with zero means and positive variance i.e$e_t^{(st)}$ $\sim\mathrm{NII}D(0, \delta^2)$ within $(s_t)(s_t)$ are random variables, assume finite values in $\{1, \dots, \mathrm{s}\}$ and serve as indicators and determined by a threshold variable $\mathrm{y_t}$ (see Tong, 1983; Tong, 1990; Hamilton, 1989). The model performances as a switching mechanism.

1.2 *The Self-Exciting Threshold Autoregressive (SETAR) Model*

One of the common forms of nonlinear time series models with threshold phenomena is a Self-Exciting Threshold Autoregressive (SETAR) model. The SETAR model is an extension of autoregressive models, which are classically fitted to time series data to allow some amount of flexibility in model parameters through regime-changing behaviours.

The SETAR models [Tong 1978, Tong, 1983] have been broadly engaged to describe several observed features of time series data (This can be found in the literature like (Watier and Richardson, 1995) on applications of epidemiology and (Clements and Smith, 2001) for water pollution. (Tong 1990) enlists various areas of application of the SETAR model while forecasting the enactment and statistical properties of the models can be found in (Tong, 1990) and (de Goojier, 2001). The Self-Exciting Threshold Autoregressive models are relatively simple to evaluate, postulate, and understand relative to other nonlinear models. A $\mathrm{SETAR}(\mathrm{d}, \mathrm{p}_i, i = 1, \dots k)$ $with$ $\mathrm{k}-$regime model is;

$$Y_t = \begin{cases} \emptyset_0^{(1)} + \sum_{j=1}^{p^{(1)}} \emptyset_j^{(1)} Y_{t-j} + e_t^{(1)}, & \text{if } Y_{t-d} \leq r_1 \\ \emptyset_0^{(2)} + \sum_{j=1}^{p^{(2)}} \emptyset_j^{(2)} Y_{t-j} + e_t^{(2)}, & \text{if } r_1 < Y_{t-d} \leq r_2 \\ \vdots & \\ \emptyset_0^{(k)} + \sum_{j=1}^{p^{(k)}} \emptyset_j^{(k)} Y_{t-j} + e_t^{(k)}, & \text{if } r_{k-1} \leq Y_{t-d} \end{cases} \quad (2)$$

where,

k is the regime's number of the SETAR model;

$p^{(j)}$is the order of the autoregressive within the j^{th} regime, $\forall j = 1, 2, \ldots, k$;

r is the threshold value $(-\infty = r_0 < r_1 < r_2 \ldots < r_{k-1} < r_k = \infty)$;

y_{t-d} is the threshold variable that runs the changes from one regime to another. The parameter (d) is a delay parameter where $(d < p)$ and d is a non-negative integer,

$e_t^{(j)}$is an *i.i.d.* white noise process with zero mean and positive constant variance, σ_j^2, $\sigma_j^2 < \infty$.

The superscripts in the model specify positions of the regime and in every regime, it is expected that the dynamical behaviors of the time series variable assume a linear autoregressive process.

Note that, if homogeneity of variance among the regimes is assumed $(\sigma_1^2 = \sigma_2^2 = \ldots = \sigma_k^2 = \sigma_e^2)$, the sample pooled variance σ_e^2 is estimated which serves as a common variance in the data. From the regime that is functioning at every time $t$is dependent on past values of $\{Y_t\}$itself, specifically, the value of Y_{t-d}. Therefore, (Tong and Lim, 1980), calls (2) a SETAR model.

1.3 *The Regime and Order Selection*

In time-series analysis, regime-switching models allow parameters to assign different values in every regime of the model. Regime-switching models are categorized into two namely; Markov-switching and threshold models. The main modification amongst these methods is how the development of the procedure is demonstrated. For example, in the threshold models, presented by (Tong, 1978), the regime shifts are generated by the status of experimental variables about some unobserved threshold variables is assumed. Sometimes the regime shifts progress according to a Markov chain, see for example (Cosslett and Lee, 1985) and (Hamilton, 1989). The models are popularly used in economic output measures, e.g Per Capital Income and total revenue, which are used to fit, identify, and forecast the stages of the commercial phase. Such models can be found in (Hamilton, 1989 and Kim et al, 2005).

2.0 Methodology

A set of data was simulated repeatedly with fixed parameter values and sample sizes under the assumption of stationarity from nonlinear autoregressive processes of second-order with trigonometric function inequation(3). This is the form of the nonlinear autoregressive processes considered for the simulation and is given by;

$$y_t = \emptyset_1 \sin(y_{t-1}) + \emptyset_2 \cos(y_{t-2}) + \varepsilon_t \quad (3)$$

where y_t the present value of the nonlinear series;

y_{t-1} and $y_{t-2}$are past values of order one and two respectively. Note that the response y_t is a nonlinear dependence on the combination of the first and second-order of past/ recent values and an 'innovation' term ε_t that introduced some new things in the model at a particular time *t* that is not described by the previous observations. The model in (3) was extracted from the idea of (Akeyede et al, 2015)

Parameters values specified for equation (3) are $\emptyset_1 = 0.7$ *and* $\emptyset_2 = -0.2$. the sample sizes that were considered in the simulation study are 20, 40, 60, 80, 100, 120, 140, 160, 180, and 200. At a particular choice of sample size, the simulation study was performed 1000 times which forms 1000 iterations for each case.

Furthermore, some steps ahead forecasting for sample sizes of 20, 100, and 200 only were also simulated, which represent low, moderate, and high sample sizes respectively to predict h-predicting steps of 5, 10, …, 50 of data generated from second-order autoregressive function. Data were simulated under stationarity assumption for response variables and error terms from;

$$Y_{ti} \sim N(0,1) and\ \varepsilon_{ti} \sim N(0,1), i = 1, \dots, 1000$$

The error term was generated from the normal distribution family of mean and variance to be zero and one respectively to ensure a white noise process in the model and therefore the data generated from these series could be stationary. SETAR (p,d) models were assessed using two criteria namely; Mean Square Error (MSE) and Akaike Information Criteria (AIC). MSE and AIC were computed for different sample sizes and forecasting of steps and models with the least criteria were considered as the best among the models.

2.1 *Forecasting of SETAR Model*

The following procedure is carried out in making a forecast with a SETAR model. Considering the observation $Y_1, Y_2, \dots, Y_t$, a forecast is made using the SETAR model by taking a weighted mean of the prediction from the 1st and 2nd regimes. At a particular time t, for a p step ahead, these forecasted values are represented by $\hat{Y}_{1,t+p}$ for the 1st regime and$\hat{Y}_{2,t+p}$ for the 2nd regime. This is presented as follows:

$$\hat{Y}_{1,t+p} = \emptyset_{0,1} + \emptyset_{1,1}\hat{Y}_{1,t+p-1,} \tag{4}$$

and

$$\hat{Y}_{2,t+p} = \emptyset_{0,2} + \emptyset_{1,2}\hat{Y}_{2,t+p-1} \tag{5}$$

The forecast of Y_{t+p}, denoted $\hat{Y}_{t+p}$, is then obtained by:

$$\hat{Y}_{t+p} = p_{p-1}\hat{Y}_{1,t+p} + (1 - p_{p-1})\hat{Y}_{2,t+p-1} + (\emptyset_{1,2} + \emptyset_{1,1})\,\hat{\sigma}_{t+p-1}\emptyset\left(\frac{c-\hat{Y}_{t+p-1}}{\hat{\sigma}_{t+p-1}}\right), \tag{6}$$

for $p = 2, \dots$ .The weight $p - 1$ is the chance of being in the lower regime at time $t + p - 1$ in the process where, $\emptyset\left(\frac{c-\hat{Y}_{t+p-1}}{\hat{\sigma}_{t+p-1}}\right)$ is assumed to be normal and $\emptyset(.)$ is the pdf of $N(0,1)$ and $\hat{\sigma}_{t+p-1}$ is the residuals' variance at time t.

Substituting$\hat{Y}_{1,t+p}\hat{Y}_{2,t+p}$, and p_{p-1}, in (6), the following recursive equation is obtained for approximate $p-$ horizon $(p > 1)$ ahead:

$$\hat{Y}_{t+p} = \emptyset\left(\frac{C-\hat{Y}_{t+p-1}}{\hat{\sigma}_{t+p-1}}\right)(\emptyset_{0,1} + \emptyset_{1,1}\hat{Y}_{t+p-1,}) + \emptyset\left(\frac{C-\hat{Y}_{t+p-1}}{\hat{\sigma}_{t+p-1}}\right)(\emptyset_{0,2} + \emptyset_{1,2}\hat{Y}_{t+p-1,}) + \emptyset\left(\frac{C-\hat{Y}_{t+p-1}}{\hat{\sigma}_{t+p-1}}\right)(\emptyset_{1,2} + \emptyset_{1,1})\hat{\sigma}_{t+p-1} \tag{7}$$

For more details, see (Clements, M. P. and Smith, 2001) and (de Goojier, and. de Bruin, 1999)

2.0 Data Analysis

The performances of SETAR (p, d), where p, d = 2, 3, i.e. SETAR (2, 2), SETAR (2, 3), SETAR (3, 2) and SETAR (3, 3) were investigated, through simulation techniques at various sample sizes.

Table 1: Results of Relative Performance of SETAR (p, d) Model at different Sample Sizes.

Sample Size (n)	MSE				AIC			
	SETAR (2, 2)	SETAR (2, 3)	SETAR (3, 2)	SETAR (3, 3)	SETAR (2, 2)	SETAR (2, 3)	SETAR (3, 2)	SETAR (3, 3)
20	0.9529	1.1211	0.9370	1.0884	3.5512	43.7319	4.1406	43.7450
40	0.9488	1.1114	0.9312	1.0739	3.3015	39.8988	3.8982	39.6670
60	0.9378	1.0976	0.9174	1.0568	2.5406	35.6485	2.9950	35.5255
80	0.9273	1.0786	0.9053	1.0318	2.2822	31.3755	2.8977	31.1396
100	0.9156	1.0560	0.8896	1.0012	2.2761	27.3449	2.7791	26.8995
120	0.8925	1.0295	0.8610	0.9631	1.4753	23.6345	1.8456	22.8809
140	0.8589	0.9777	0.8183	0.8948	0.7010	18.8351	0.7177	17.6305
160	0.8175	0.9045	0.7626	0.8028	0.6712	14.5057	0.4310	13.1912
180	0.7241	0.7448	0.6436	0.5955	-0.2483	8.4707	-1.1195	5.1741
200	0.4700	0.5198	0.2427	0.0676	-2.8632	8.4652	-10.3487	-25.8792

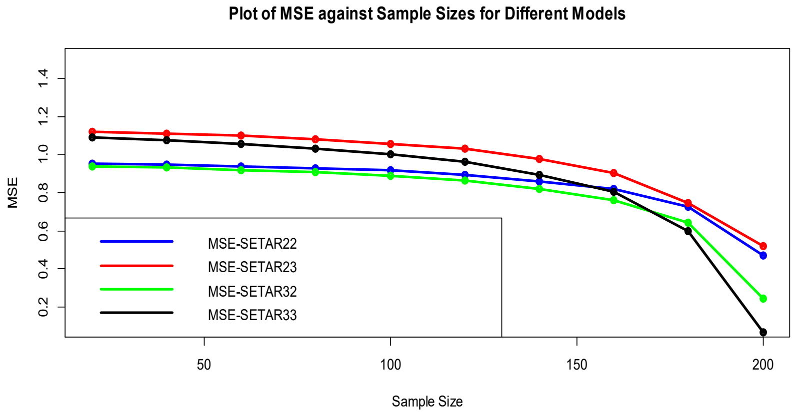

Fig. 1a: The plots ofMSE of SETAR (p, d) Model at different Sample Sizes

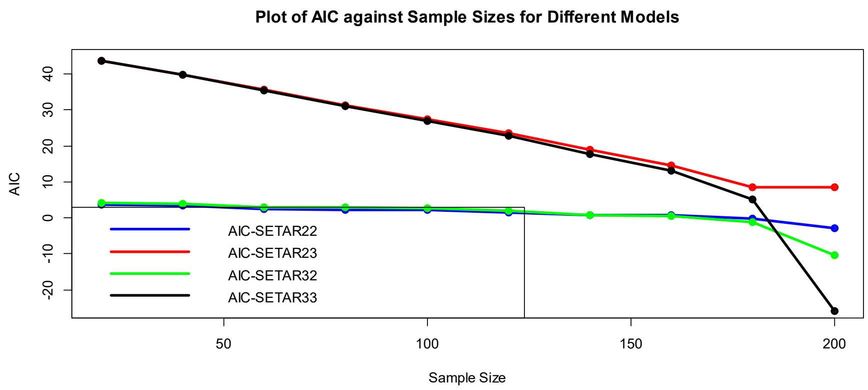

Fig. 1b: The plots of AIC of SETAR (p, d) Model at different Sample Sizes.

The best order (p) and regime number (d) were determined under the SETAR (p, d) model in fitting and forecasting the simulated nonlinear autoregressive model in (3). The results of the analyses are provided in tables and presented on graphs as shown as follows.

The average values of MSEs and AICs of each model at various sample sizes were recorded in table 1 and presented in figures 1a and 1b respectively. The best model to fit the nonlinear model is SETAR (3, 2) from a sample size of 20 to 180, followed by SETAR (2, 2) based on both MSE and AIC criteria. However, SETAR (3, 3) outperforms others from a sample size of 180 and above (large sample sizes).

The worst model was observed from SETAR (2, 3)at all sample sizes. Hence the best autoregressive and regime orders to be selected for fitting nonlinear autoregressive time series data with small and moderate sample sizes are 3rd and 2nd autoregressive and regime orders respectively while those with large sample sizes can be fitted with 3rd autoregressive and regime orders respectively. Also, it was observed from figures 1a and 1b that there are increments in the performances of all the models when the sample size is improved, this was shown from a decrease in values of MSE and AIC as sample size increases.

3.1 *ForecastPerformances of SETAR (p, d) Models on Nonlinear Autoregressive Data*

Comparison of forecast ability among the four fitted models to simulated nonlinear autoregressive model when $20, 100\ and\ 200$ (small, moderate, and large) sample sizes were used for simulations. The outcomes achieved were presented in figures 2a and 2b respectively.

Table 2: Results of Relative Forecast Performance of SETAR (p, d) Model at Sample Size 20.

Steps Ahead (h)	**MSE**				**AIC**			
	SETAR (2, 2)	SETAR (2, 3)	SETAR (3, 2)	SETAR (3, 3)	SETAR (2, 2)	SETAR (2, 3)	SETAR (3, 2)	SETAR (3, 3)
5	0.6476	0.3823	7555.7600	4953.737	0.1302	-7.4221	4.1095	285.8044
10	0.0004	0.2805	3098.6280	173.4843	-794.008	-8.1663	-5.0270	147.9873
15	0.0002	0.0028	596.9272	50.6988	-1143.25	-1139.450	-6.4755	47.255
20	1.23E-05	6.59E-23	6.5361	11.0204	-1325.35	-1409.370	-500.4990	-61.2087
25	8.64E-14	4.25E-32	0.4206	1.5340	-1502.38	-1815.570	-1065.690	-110.4620
30	6.41E-33	1.87E-33	0.2092	0.2589	-1883.62	-2280.250	-1085.020	-124.9340
35	6.01E-33	1.83E-33	6.81E-05	0.0116	-2011.42	-2658.710	-1131.670	-778.7800
40	2.18E-33	1.58E-33	1.52E-31	0.0005	-2719.69	-3036.880	-1551.560	-1231.020
45	3.12E-34	1.39E-33	8.88E-33	6.38E-26	-3024.11	-3069.370	-1596.150	-1422.360
50	1.96E-35	1.14E-33	6.77E-35	4.58E-28	-3152.83	-3419.260	-2177.550	-1605.840

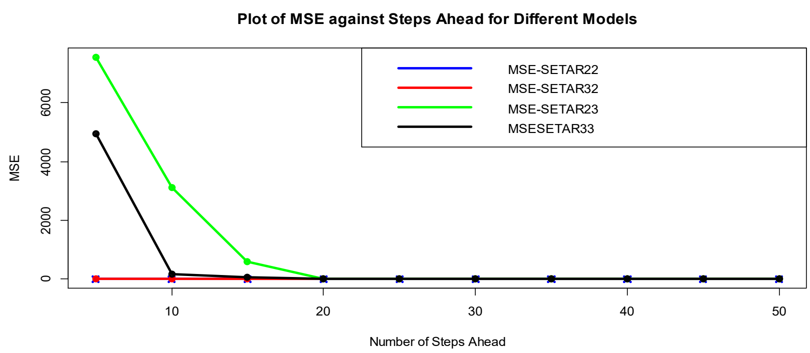

Fig. 2a: The plots of MSE of Forecast Performances of SETAR (p, d) Model at Sample Size 20.

Table 2 shows the means of MSEs and AICs obtained from forecasting values of each model at a sample size of 20. The best forecast model is SETAR (3, 2) from steps ahead of 5 to 45 especially based on AIC, and have a very close performance with SETAR (2,2) especially at small and moderate steps ahead (*h*). Moreso, the four models forecast nearly equal when *h* rises. Their forecast performance increases as the steps ahead increase when the sample size is 20.

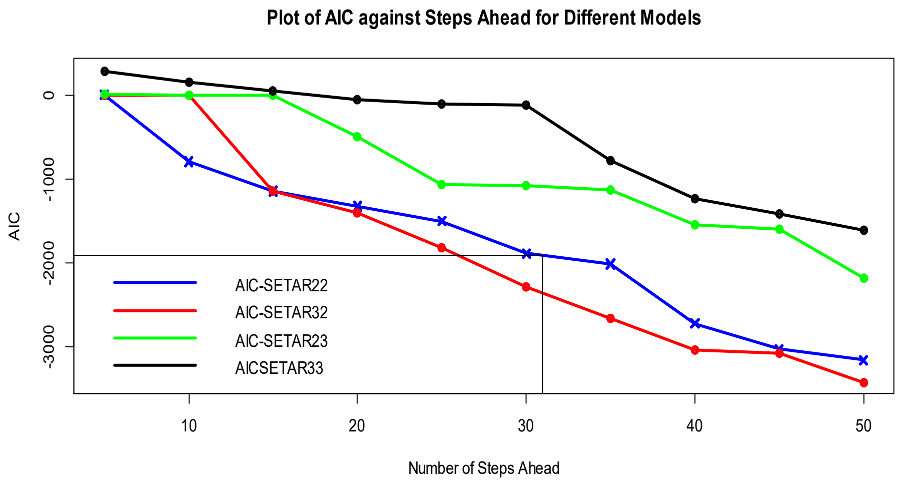

Fig. 2b: The plots of AIC Forecast Performance of SETAR (p, d) Model at Sample Size 20.

Given the above results, the best autoregressive and regime orders to be selected for forecasting nonlinear autoregressive time series data with small and moderate steps ahead are 3rd& 2nd and 2nd& 2nd autoregressive and regime orders respectively. However, from moderate to large steps ahead any autoregressive and regime orders can be selected for forecasting nonlinear autoregressive time series data.

Table 3: Results of Relative Forecast Performance of SETAR (p, d) Model at Sample Size 100.

Steps Ahead (*h*)	MSE				AIC			
	SETAR (2, 2)	SETAR (2, 3)	SETAR (3, 2)	SETAR (3, 3)	SETAR (2, 2)	SETAR (2, 3)	SETAR (3, 2)	SETAR (3, 3)
5	0.9248	0.8930	3.3361	324352.5	-10.8086	27.8947	43.0048	30.2536
10	5.14E-08	0.8339	1.1072	6407.352	-780.779	5.5255	22.8250	19.7270
15	9.88E-34	1.75E-33	0.7781	492.2123	-1154.91	-1145.710	14.1449	4.8300
20	9.72E-34	1.72E-33	0.2885	138.5937	-1515.64	-1554.650	0.1240	-736.5140
25	8.27E-34	1.47E-33	0.0306	0.9753	-1520.04	-1913.210	-803.8830	-1154.070
30	2.2E-34	8.77E-34	0.0039	0.8231	-2301.89	-2260.300	-1063.570	-1392.130
35	9.07E-35	5.42E-34	0.00389	0.7550	-2684.77	-2637.640	-1136.330	-1461.500
40	8.58E-35	2.03E-34	4.4E-06	0.04858	-3123.85	-3054.820	-1365.450	-1717.410
45	8.54E-35	1.09E-34	-1305.490	0.0122	-3602.56	-3452.140	-1859.950	-2103.140
50	1.63E-35	7.08E-35	-1330.880	0.0003	-3991.13	-3891.980	-2743.700	-2195.300

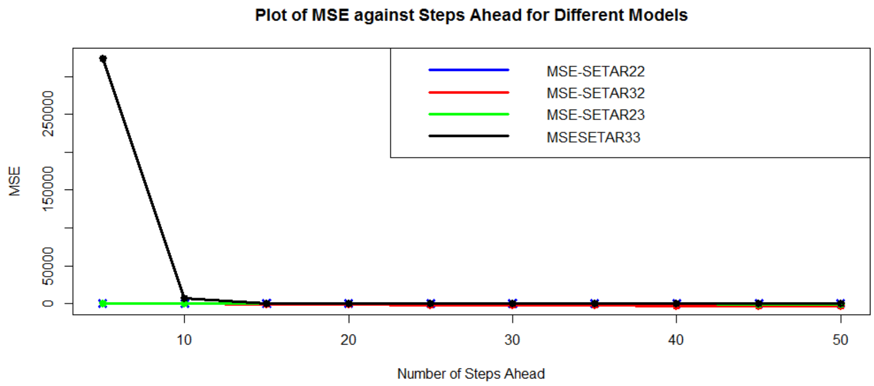

Fig. 3a: The plots of MSE Forecast Performance of SETAR (p, d) Model at Sample Size 100.

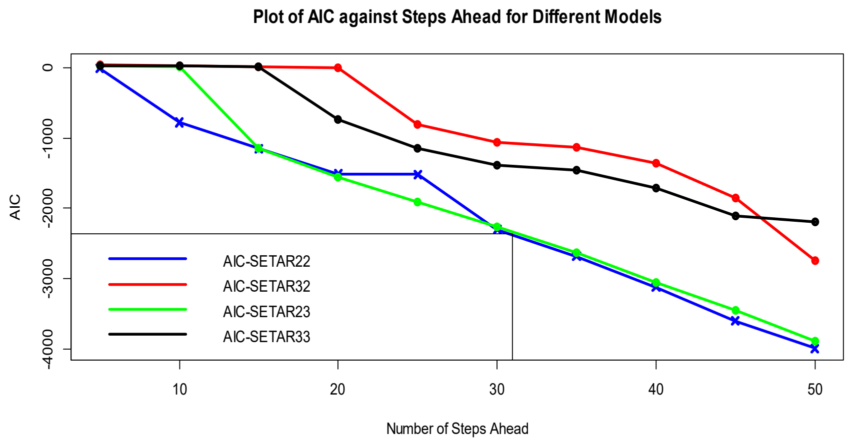

Fig. 3b: The plots of AIC Forecast Performance of SETAR (p, d) Model at Sample Size 100.

Table 3 shows the means of MSEs and AICs obtained from forecasting values of each model at a sample size of 100. From the fig 3a and 3b, it was observed that SETAR (2, 3) and SETAR (2, 2) have the best forecast, as shown by the two criteria, at all step ahead levels followed by SETAR (2, 2) and SETAR (2, 3) respectively.

The worst forecast models are SETAR (3, 3) based on MSE and SETAR (3, 2) based on AIC. Hence, the best autoregressive and regime orders to be selected for forecasting nonlinear autoregressive time series data at any level of steps ahead and sample size of 100 are 2^{nd}, 3^{rd}, and 2^{nd}, 2^{nd} autoregressive and regime orders respectively.

Table 4 shows the average values of MSEs and AICs obtained from forecasting values of each model at a sample size of 200. From the fig 4a and 4b, it was observed that SETAR (2, 2) has the best forecast based on MSE from steps ahead of 5 and 10 (lower steps forecast) based on MSE. SETAR (3, 2) performs better than others from steps ahead of 10 to 50.

Table 4: Results of Relative Forecast Performance of SETAR (p, d) Model at Sample Size 200.

Steps Ahead (h)	MSE				AIC			
	SETAR (2, 2)	SETAR (2, 3)	SETAR (3, 2)	SETAR (3, 3)	SETAR (2, 2)	SETAR (2, 3)	SETAR (3, 2)	SETAR (3, 3)
5	0.9356	0.9817	1.0883	31.8660	24.1453	16.2064	50.0075	49.7832
10	0.8652	0.9182	0.8735	4.1824	-14.9909	-40.7843	49.7913	33.0097
15	3E-33	2.09E-33	0.3892	1.2028	-1153.28	-1191.900	-1130.250	16.7305
20	2.76E-33	1.57E-33	0.3048	0.9757	-1539.38	-1522.860	-1151.850	-1481.800
25	2.69E-33	7.8E-34	0.0197	0.9531	-1915.21	-1906.910	-1319.040	-1531.590
30	1.17E-33	6.26E-34	0.0049	0.2659	-2264.44	-2312.910	-1433.500	-1772.700
35	6.24E-34	5.97E-34	0.0011	0.1329	-2651.75	-2700.490	-1688.030	-1822.560
40	6.04E-34	2.67E-34	0.0007	0.0001	-3045.56	-3078.180	-1969.350	-1943.300
45	4.49E-34	2E-34	0.0002	4.53E-06	-3422.23	-3465.470	-2196.520	-1972.010
50	1.86E-34	3.2E-35	1.03E-34	6.19E-34	-3777.24	-3744.38	-2705.330	-2577.360

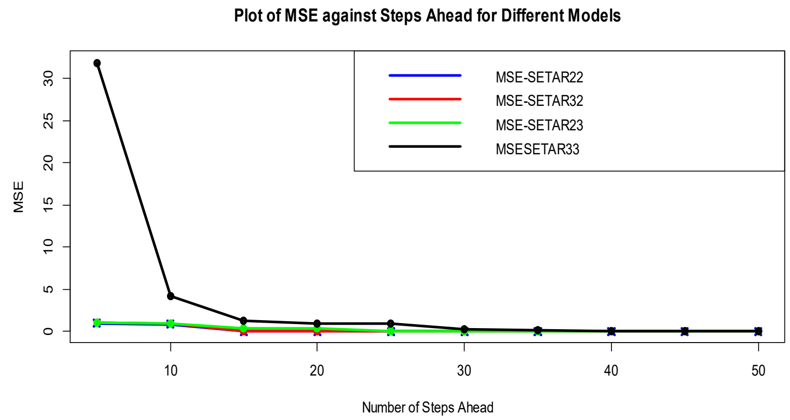

Fig.4a: The plots of MSE Forecast Performance of SETAR (p, d) Model at Sample Size 200.

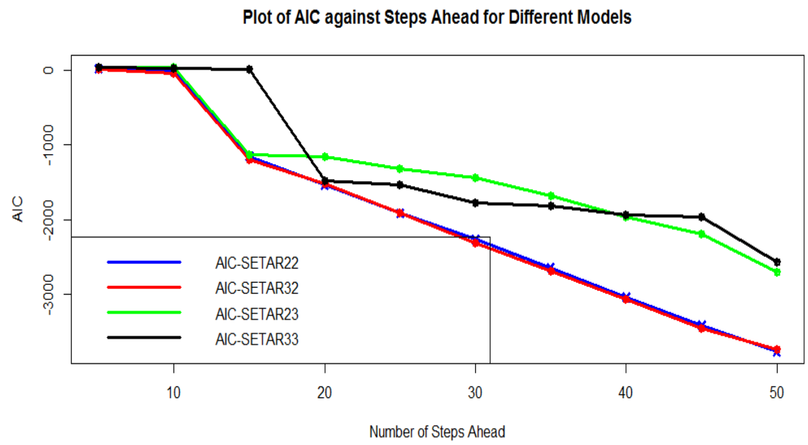

Fig.4b: The plots of AIC Forecast Performance of SETAR (p, d) Model at Sample Size 200.

The three other models competed well with SETAR (3, 2) in forecasting 50 steps ahead (highest steps ahead) based on MSE. Furthermore, based on AIC, SETAR (3, 2) has the best forecast at all levels of steps ahead followed by SETAR (2, 2).

The best autoregressive and regime orders to be selected for forecasting nonlinear autoregressive time series data at any level of steps ahead and sample size of 200 are 2nd and 2nd followed by 3rd and 2nd based on MSE and 3rd and 2nd followed by 2^{nid} and 2nd in respect to AIC respectively. The forecasting ability of the four models increased when the steps ahead are increased.

3.0 Conclusion

The best autoregressive and regime orders to be selected for fitting nonlinear autoregressive time series data with small and moderate sample sizes are 3rd and 2nd autoregressive and regime orders respectively while those with large sample sizes can be fitted with 3rd autoregressive and regime orders respectively. The performances of the four models increased when the sample increased, with a minimum value of MSE and AIC.

Furthermore, the best autoregressive and regime orders to be selected for forecasting nonlinear autoregressive time series data with lower and moderate steps ahead and sample size of 20 are combinations of 3rd, 2nd, and 2nd, 2nd autoregressive and regime orders respectively. However, from moderate and higher steps ahead any autoregressive and regime orders can be selected for forecasting nonlinear autoregressive time series data.

Moreso, for any level of steps ahead and sample sizes of 100, the best autoregressive and regime orders to be selected for forecasting nonlinear autoregressive time series data are 2nd autoregressive order and 2nd regime orders. So also, at any level of steps ahead and sample size of 200, the best autoregressive and regime orders to be selected for forecasting nonlinear autoregressive time series data are 3rd and 2nd followed by 2ndand 2nd.

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