Annals of Statistical Theory and Applications (ASTA)
Vol.1, 31-41, May, 2019

Original Research Article

Reduced Beta Skewed Laplace Distribution with Application to Failure-Time of Electrical Component Data

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Received: 10th March 2018 *Revised: 21st April 2019* *Accepted: 27th April 2019*

Abstract

In this work, a new skewed distribution called the reduced beta skewed Laplace distribution was proposed using the T-$R\{Y\}$ method. The T-$R\{Y\}$ nomenclature is about the newest methods of generating families of probability distributions. This method is very important in survival analysis in that each generated distribution is considered as a weighted hazard function of the base random variable, R. Some structural properties of the proposed distribution were derived, such as expressions for its moments, moments of the order statistics, and so forth. The maximum likelihood estimation of the model parameters was discussed, and the observed information matrix was derived. The importance and usefulness of the proposed model were illustrated using a real data set on time to fail of electrical components. Results from the Monte-Carlo experiment are quite good in favour of the new distribution. The performance of the proposed distribution was compared with those of other known selected distributions based on the real-life dataset and the results showed a good performance of the new model relative to others. This new distribution will provide new opportunities for assessing reliability and survival data in medicine, health, finance, environment, military, and other areas.

Keywords: Reduced beta distribution, Skewed Laplace distribution, T-R{Y} framework, Survival function, Maximum likelihood estimation.

1.0 Introduction

Continuous probability distributions are commonly applied to describe real-world phenomena. Due to the usefulness of these distributions, their theory is widely studied and new distributions are developed. Developing a new probability distribution from existing known distributions is a common scenario in the field of probability distribution. Generalized classes of probability distributions have been proposed and applied to describe various phenomena even in the areas of environmental hazards. The probability distribution in the family of distribution that will be developed will be very useful in modeling environmental hazard data, reliability and survival data.

Recently, some newly proposed probability distributions and their applications had been published in the literature. See Famoye et al. (1998, 2004, 2005), Lee et al. (2013), Alzaatreh el al. (2013, 2014), Eugene et al. (2002) and Aljarrah et al. (2014) all developed new probability distributions and some of them were applied to various aspect of real-life phenomena.

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Cordeiro and de Castro (2011) studied the properties and application of Kumaraswamy – G distribution while Alizadeh et al. (2014) developed the family of continuous probability distributions referred to as the Kumaraswamy Marshal-Olkin generalized family of distributions.

A new method for generating families of continuous distributions called T-X family by Alzaatreh et al. (2013) was introduced by replacing the beta PDF with a PDF, $r(t)$, of a continuous random variable and applying a function $W(F(x))$ that satisfies some specific conditions.

Aljarrah et al. (2014) used quantile functions to generate T-X{Y} family of distributions.

$$G(x) = \int_{a}^{W[F(x)]} r(t)dt = R\{W[F(x)]\}$$

Alzaatreh et al. (2016) introduced the family of generalized Cauchy distributions, T-Cauchy{Y} family, using the T-R{Y} framework.

This paper is organized as follows. The proposed distribution is derived in Section 2. Its properties are provided in Section 3 such as the shapes of the density, distribution, hazard, survival and quantile functions, related distributions, raw moment and maximum likelihood estimation (MLE). Result using one application of real data sets to illustrate the potentiality of the new distribution in Section 4. The paper is concluded in Section 5.

2.0 Transformed-Transformer Method (*T-R*{*Y*})

Alzaatreh et al (2014) in their work used different distributions other than the Beta distribution (see Eugene et al, 2002) with supports different from [0, 1] as a generator to derive different classes of distributions. Alzaatreh et al. (2014) developed a general method, which is an improvement to the beta-generated family that allows for using any continuous PDF as the generator. This new method is called the transformed-transformer method (*T-R*{*Y*}). Let $f_T(x)$ be the PDF of a random variable $T\in[a,b]$, $-\infty \le a<b \le \infty$. Let $F_R(x)$ be a CDF of any random variable R and $Q_Y(x)$ is the quantile function of any random variable so that $Q_Y(F_R(x))$ satisfies the following conditions:

$Q_Y(F_R(x)) \in [a,b]$, $Q_Y(F_R(x))$ is differentiable and monotonically non- decreasing, $Q_Y(F_R(x)) \to a$ as $x\to -\infty$ and $Q_Y(F_R(x)) \to b$ as $x\to\infty$. The CDF of a new family of distributions is defined as

$$F_Z(x) = \int_{a}^{Q_Y[F_R(x)]} f_T(t)dt = P\{T \le Q_Y[F_R(x)\} = F_T\{Q_Y[F_R(x)]\}\,. \tag{1}$$

Where $Q_Y(F_R(x))$ satisfies the conditions in equation (2) and $F_T(x)$ is the CDF of the random variable T. The corresponding PDF is

$$f_Z(x) = \left\{\frac{d}{dx} Q_Y[F_R(x)]\right\} f_T\{Q_Y[F_R(x)]\} \tag{2}$$

Equations (1) and (2) are the CDF and PDF of the T-R{Y} family of distributions, which is a generalization of the transformed-transformer family. The new PDF $f_Z(x)$ is the PDF transformed from the random variable T through the 'transformer' random variable Y using the quantile function of a random variable R. So, $Z = f(T,R,Y)$.

3.0 Derivation of Reduced Beta Laplace Distribution

The CDF $F(x)$ for the reduced Beta distribution with $b = 1$ and $a > 0$, is given by

$$F(x) = x^a, \qquad a > 0 \tag{3}$$

This is also the same as Kumaraswamy distribution with $b = 1$.
The mean of the reduced Beta distribution is $\frac{a}{a+1}$ and the variance is $\frac{a}{(a+2)(a+1)^2}$.
Let $f(x)$ be the pdf of Laplace distribution given by

$$f(x;\mu,b) = \frac{1}{2b}e^{\left(-\frac{|x-\mu|}{b}\right)}, x \in (-\infty,\infty) \tag{4}$$

and CDF, $F(x)$ is given as:

$$F(x;\mu,b) = \begin{cases} \frac{1}{2}e^{\left(-\frac{x-\mu}{b}\right)}, & if\ x < \mu \\ 1-\frac{1}{2}e^{\left(-\frac{x-\mu}{b}\right)} & if\ x \geq \mu \end{cases} \tag{5}$$

It is expected that the quantity to be measured by the distribution to be derived is non-negative. Most of the real-life quantities to be measured are non-negative values. With the assumption that the data, which the new distribution will attempt to model are non-negative, it is, therefore, necessary to reduce the Laplace distribution to positively skewed distribution.

Thus, the positive skewed Laplace distribution is given by

$$f(x;\mu,b) = \frac{1}{b}e^{\left(-\frac{x-\mu}{b}\right)}, x > \mu \geq 0; b > 0 \tag{6}$$

and the CDF of the skewed Laplace distribution is given by

$$F(x) = 1 - e^{\left(-\frac{x-\mu}{b}\right)}, \ x > \mu \geq 0; b > 0 \tag{7}$$

The skewed Laplace distribution reduces to the exponential distribution when $\mu = 0$. The new PDF is now the combination of the reduced beta distribution and skewed Laplace distribution, using the T-$R\{Y\}$ framework. The CDF of the reduced beta skewed Laplace variable, X is derived as

$$F_Z(x) = \int_0^{Q_Y(F_R(x))} f_T(t)dt \tag{8}$$

where $f_T(t)$ is the PDF of the reduced beta random variable, $Q_Y()$ is the quantile function of a uniform distribution $F_R(x)$ is the CDF of a skewed Laplace distribution and $F_Z(x)$ is the CDF of the new distribution, that is, the reduced beta skewed Laplace. So, from (18) we have

$$F_Z(x) = \int_0^{1-e^{\left(-\frac{x-\mu}{b}\right)}} at^{a-1}\,dt = [t^a]_0^{1-e^{\left(-\frac{x-\mu}{b}\right)}} \tag{9}$$

$$F_Z(x) = \left[1-e^{\left(-\frac{x-\mu}{b}\right)}\right]^a, \qquad x > \mu \geq 0; b > 0, a > 0 \tag{10}$$

The PDF corresponding to the reduced beta skewed Laplace distribution is derived as follows

$$f_Z(x) = \frac{dF_Z(x)}{dx} = \frac{d\left[1-e^{\left(-\frac{x-\mu}{b}\right)}\right]^a}{dx}; \ x > \mu \geq 0; b > 0, a > 0 \tag{11}$$

$$f_Z(x) = \left(\frac{a}{b}\right)\left[e^{\left(-\frac{x-\mu}{b}\right)}\right]\left[1 - e^{\left(-\frac{x-\mu}{b}\right)}\right]^{a-1} \; ; \; x > \mu \geq 0; b > 0, a \geq 1 \qquad (12)$$

where$\mu \geq 0$ is the location parameter, $b > 0$ is the scale parameter. Both μ and b are parameters from Laplace distribution, while $a > 0$ is the shape parameter from the reduced beta skewed Laplace distribution.

In summary, the PDF, f_Z(x) and the CDF, $F_Z(x)$ of the new probability distribution are given below as

$$f_Z(x) = \left(\frac{a}{b}\right)\left[e^{\left(-\frac{x-\mu}{b}\right)}\right]\left[1 - e^{\left(-\frac{x-\mu}{b}\right)}\right]^{a-1} ; x > \mu \geq 0; b > 0; a > 0$$

and

$$F_Z(x) = \left[1 - e^{\left(-\frac{x-\mu}{b}\right)}\right]^{a} ; \; x > \mu \geq 0; b > 0; a > 0$$

The quantile function is given by

$$x = \mu - bln\left[1 - p^{\frac{1}{a}}\right]; \; \mu \geq 0; b > 0; a > 0$$

where p is a random variate simulated from a uniform distribution. The new distribution is supported below by μ and above by ∞.

3.1 Properties of Reduced Beta Skewed Laplace Distribution

3.1.1 Cumulative Distribution Function (CDF)

Equation (10) is now the CDF of the new probability distribution called reduced beta skewed Laplace distribution.

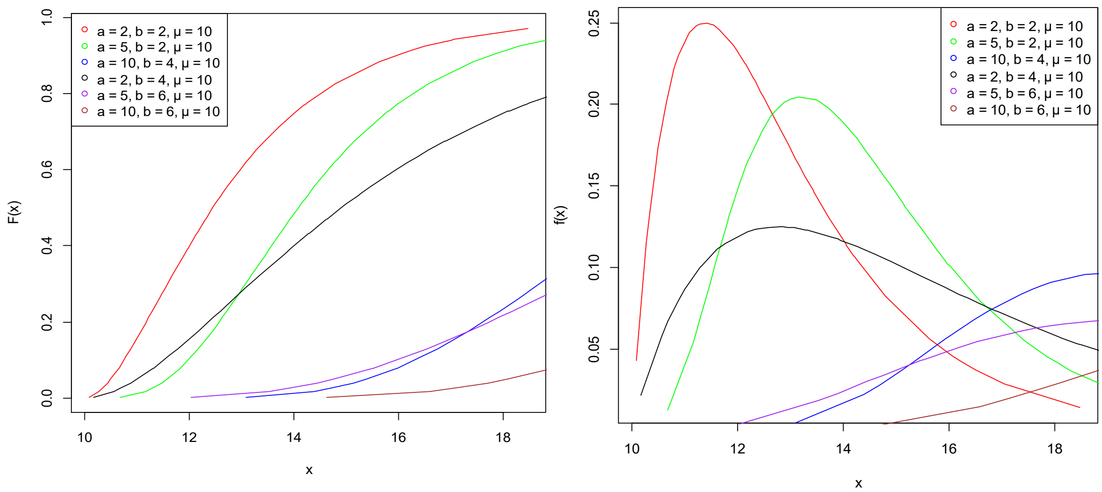

Figure 1: CDFs and PDF of reduced beta skewed Laplace distribution for $\mu = 10$ and for various values of a and b

3.1.2 Probability Density Function (PDF)

The PDF of Reduced beta skewed Laplace probability distribution with three parameters is given in (12). It is important we prove that Reduced beta skewed Laplace distribution is a proper PDF. A PDF must satisfy that

$$\int f(x)\,dx = 1 \tag{13}$$

Proof: To show that

$$\int_{\mu}^{\infty}\left(\frac{a}{b}\right)\left[e^{\left(-\frac{x_i-\mu}{b}\right)}\right]\left[1-e^{\left(-\frac{x_i-\mu}{b}\right)}\right]^{a-1} dx = 1 \tag{14}$$

We have to integrate the left hand side of (23) by substation method

$$\int_{\mu}^{\infty}\left(\frac{a}{b}\right)\left[e^{\left(-\frac{x_i-\mu}{b}\right)}\right]\left[1-e^{\left(-\frac{x_i-\mu}{b}\right)}\right]^{a-1} dx$$

Let $w = 1 - e^{\left(-\frac{x_i-\mu}{b}\right)}$, so that $e^{\left(-\frac{x_i-\mu}{b}\right)} = 1 - w$ and $\frac{dw}{dx} = \frac{1}{b}e^{\left(-\frac{x_i-\mu}{b}\right)}$;

$$dx = \frac{bdw}{e^{\left(-\frac{x_i-\mu}{b}\right)}} = \frac{bdw}{1-w}$$

$$\frac{a}{b}\int_{\mu}^{\infty}[1-w][w]^{a-1}\frac{bdw}{1-w} = a\int_{\mu}^{\infty}[w]^{a-1}\,dw = |w^{a}|_{\mu}^{\infty} \tag{15}$$

But $w = 1 - e^{\left(-\frac{x_i-\mu}{b}\right)}$ so that we substitute *w*in (13) to obtain the desired result.

$$\left|\left(1-e^{\left(-\frac{x_i-\mu}{b}\right)}\right)^{a}\right|_{\mu}^{\infty} = 1$$

Figure 1 presents the graphs of the CDF and PDF of Reduced beta skewed Laplace distribution respectively for different parameter values. Both graphs show different curves for various values of the parameter a and b for a constant value of μ. Figure 1b shows that Reduced beta skewed Laplace distribution is a positively skewed distribution.

3.1.3 Hazard Function

The hazard function of the Reduced beta skewed Laplace distribution is derived from this definition

$$h(x) = \frac{f_Z(x)}{1-F_Z(x)}$$

where$f_Z(x)$ and $F_Z(x)$ are the PDF and CDF of reduced beta skewed Laplace distribution given in (12) and (10) respectively. The hazard function $h(x)$ can be written as

$$h(x) = \frac{\left(\frac{a}{b}\right)\left[e^{\left(-\frac{x-\mu}{b}\right)}\right]\left[1-e^{\left(-\frac{x-\mu}{b}\right)}\right]^{a-1}}{1-\left[1-e^{\left(-\frac{x-\mu}{b}\right)}\right]^{a}} \tag{16}$$

3.1.4 Survival Function, S(x)

The S(x) of the Reduced beta skewed Laplace distribution is derived from this definition

$$S(x) = 1 - F_Z(x)$$

where $F_Z(x)$ is the CDF of Reduced beta skewed Laplace distribution given in (10) . The survival function S(x) can be written as

$$S(x) = 1 - \left[1-e^{\left(-\frac{x-\mu}{b}\right)}\right]^{a} \tag{17}$$

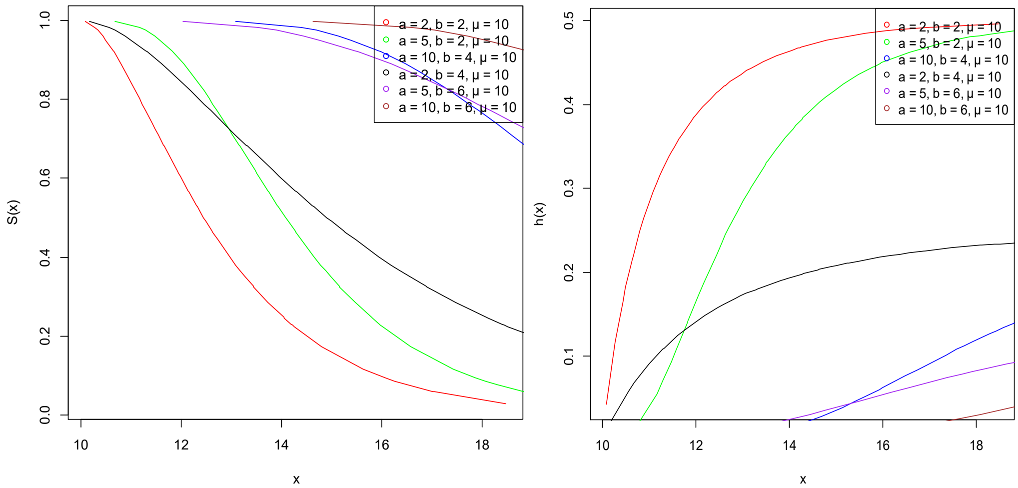

Figure 2: (a)Hazard function and (b)Survival rates of Reduced beta skewed Laplace distribution for μ = 10 and for various values of a and b

Figure 2a depicts the curves of the corresponding hazard functions for the PDFs in Figure 1b of Reduced beta skewed Laplace distribution for a constant value of μ and for various values of a and b. As the x increases, the hazard function also increases up to a level and remains constant but is not decreasing.

Figure 2b depicts the curves of the corresponding survival functions for the PDFs in Figure 1b of Reduced beta skewed Laplace distribution for a constant value of μ and for various values of a and b. As x increases, the survival function decreases steadily from 1 to 0. This is a typical survival curve of any survival distribution.

3.1.5 Cumulative Hazard Function, H(x)

The, $H(x)$ of the Reduced beta skewed Laplace distribution is derived from this definition

$$H(\mathrm{x}) = -log_e S(x)$$

where $S(x)$ is the survival function of Reduced beta skewed Laplace distribution given in (17). It can be written as

$$H(\mathrm{x}) = -log_e\left(1 - \left[1 - e^{\left(-\frac{x-\mu}{b}\right)}\right]^a\right) \tag{18}$$

3.1.6 Quantile Function (Q(x)) and Measures of Partition

The Q(x) of Reduced beta skewed Laplace distribution is the inverse function of the CDF, which was derived in (13). Recall from (13)

$$x = \mu - bln\left[1 - p^{\frac{1}{a}}\right];\ \ \mu \geq 0; b > 0; a > 0$$

(i) The median of Reduced beta skewed Laplace distribution is given by

$$\tilde{x} = Q_X(p) = Q_X(0.5) = \mu - bln\left[1 - (0.5)^{\frac{1}{a}}\right]$$

(ii) The 1st quartile, Q_1 of the Reduced beta skewed Laplace distribution is given by

$$Q_X(0.25) = \mu - bln\left[1 - (0.25)^{\frac{1}{a}}\right]$$

(iii) The 3rd quartile, Q_3 of the Reduced beta skewed Laplace distribution is given by

$$Q_X(0.75) = \mu - bln\left[1 - (0.75)^{\frac{1}{a}}\right]$$

(iv) The minimum, $x_{(1)}$ of the Reduced beta skewed Laplace distribution is given by$x_{(1)} = \mu$.

(v) The maximum, $x_{(n)}$ of the Reduced beta skewed Laplace distribution is given by$x_{(n)} = \mu + A$. Where A is a positive real number such that A is a function of the product of b and any positive real number.

Let x_i (i = 1 to n) be a set of n random observations, then $x_{(i)}$ is a set of ordered observations. The minimum and maximum values are $x_{(1)}$ and$x_{(n)}$respectively.

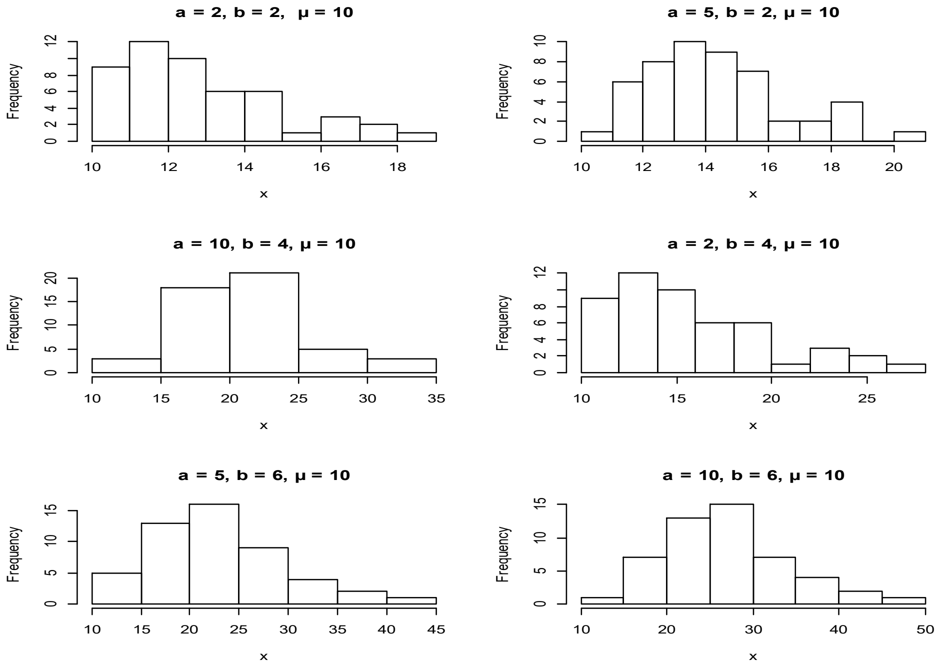

Figure 3: Histogram of Reduced beta skewed Laplace distribution for μ = 10 and various values of a and b

Figure 3 depicts six different histograms depicting the different shapes of Reduced beta skewed Laplace distribution for the constant value of μ and for various values of a and b. The different shapes show that Reduced beta skewed Laplace distribution is a positively skewed distribution and has a tendency of becoming normal for some values of a and b. It also has the tendency of becoming bimodal for some values of a and b.

3.1.7 Moment about the origin

The rth moment about the origin of the Reduced beta skewed Laplace distribution is given as

$$\mu_r^{`} = \frac{a}{b}\sum_{j=0}^{a-1}\sum_{i=0}^{\infty}\binom{a-1}{j}\binom{r}{i}(-1)^j\mu^i\frac{\Gamma(r-i+1)}{\left(\frac{2}{b}\right)^{(r-i+1)}}$$

Proof:

$$\grave{\mu_r} = \int_{\mu}^{\infty} x^r f_Z(x)dx$$

where $f_Z(x)$ is the pdf of Reduced beta skewed Laplace distribution as given in (12)

$$\grave{\mu_r} = \int_{\mu}^{\infty} x^r \left(\frac{a}{b}\right) \left[e^{\left(-\frac{x-\mu}{b}\right)}\right] \left[1 - e^{\left(-\frac{x-\mu}{b}\right)}\right]^{a-1} dx$$

By using binomial expansion

$\left[1 - e^{\left(-\frac{x-\mu}{b}\right)}\right]^{a-1} = \sum_{j=0}^{a-1} \binom{a-1}{j} (-1)^j e^{\left(-\frac{x-\mu}{b}\right)}$ then

$$\grave{\mu_r} = \int_{\mu}^{\infty} x^r \left(\frac{a}{b}\right) \left[e^{\left(-\frac{x-\mu}{b}\right)}\right] \sum_{j=0}^{a-1} \binom{a-1}{j} (-1)^j e^{\left(-\frac{x-\mu}{b}\right)} dx$$

Let $y = x - \mu 0 \leq y \geq \infty$ then $x= y+ \mu$

$$\therefore dx = dy$$

$$\grave{\mu_r} = \left(\frac{a}{b}\right) \sum_{j=0}^{a-1} \binom{a-1}{j} (-1)^j \int_{0}^{\infty} (y + \mu)^r e^{\left(-2\frac{y}{b}\right)} dy$$

$$A = (y + \mu)^r = \sum_{i=0}^{\infty} \binom{r}{i} y^{r-i} \mu^i$$

Using power series and integrate the above equation, we have

$$\grave{\mu_r} = \frac{a}{b} \sum_{j=0}^{a-1} \sum_{i=0}^{\infty} \binom{a-1}{j} \binom{r}{i} (-1)^j \mu^i \frac{\Gamma(r - i + 1)}{\left(\frac{2}{b}\right)^{(r-i+1)}}$$

3.2 Related Distributions

(i) When $a = 1$ and $\mu = 0$, Reduced beta skewed Laplace distributions becomes **exponential** distribution

$$f(x) = \left(\frac{1}{b}\right) \left[e^{\left(-\frac{x}{b}\right)}\right]; x > 0; b > 0$$

(ii) When $a = 1$ and μ takes on any real value, Reduced beta skewed Laplace distributions becomes **Laplace** distribution

$$f(x) = \left(\frac{1}{b}\right) e^{-\frac{|x-\mu|}{b}}; 0 < x < \infty; b > 0$$

(iii) When b= 1/λ and x-μ = z, Reduced beta skewed Laplace distribution becomes the **Exponentiated Weibull** distribution with γ =1,

$$f(z) = \alpha\lambda\left[e^{(-\lambda z)}\right]\left[1 - e^{(-\lambda z)}\right]^{a-1}; z > 0 \text{ or or } x - \mu \geq 0$$

Unimodality

Reduced beta skewed Laplace distribution is unimodal. If the mixing (Transformed) distribution is nonnegative, continuous, and unimodal then the resulting distribution is unimodal (Holgate, 1970). Thus, the proposed model is unimodal since the transformed one-parameter-Kumaraswamy distribution is unimodal.

Theorem 3.1: The Reduced beta skewed Laplace model (12) is unimodal for all values of (a, b, μ) and the modal function is given as:

$$x = \mu + blna \; ; \; x > \mu \geq 0; b > 0; a > 0$$

Proof:

The prove is given.

3.3 Maximum Likelihood Estimation

A random sample is taken from the Reduced beta skewed Laplace distribution with probability density function (PDF) given as

$$f(x) = \left(\frac{a}{b}\right)\left[e^{\left(-\frac{x_i-\mu}{b}\right)}\right]\left[1 - e^{\left(-\frac{x_i-\mu}{b}\right)}\right]^{a-1} \; ; x > \mu \geq 0; b > 0; a > 0$$

and the log-likelihood function is given by:

$$\text{LnL}(x; a, b, \mu) = nlna - nlnb - \frac{1}{b}\sum_{i=1}^{n} x_i + \frac{n\mu}{b} + (a-1)\sum_{i=1}^{n} \ln\left(1 - e^{\left(-\frac{x_i-\mu}{b}\right)}\right) \tag{19}$$

Taking a partial differentiation of equation (19) with respect to a, b and μ respectively

Let Q = LnL(x;a,b,μ)

$$\frac{\partial Q}{\partial a} = \frac{n}{a} + \sum_{i=1}^{n} \ln\left(1 - e^{\left(-\frac{x_i-\mu}{b}\right)}\right) \tag{20}$$

$$\frac{\partial Q}{\partial b} = -\frac{n}{b} - \frac{1}{b^2}\sum_{i=1}^{n} x_i - \frac{n\mu}{b^2} - \frac{(a-1)}{b^2}\sum_{i=1}^{n}\left(\frac{(x_i - \mu)e^{\left(-\frac{x_i-\mu}{b}\right)}}{1 - e^{\left(-\frac{x_i-\mu}{b}\right)}}\right) \tag{21}$$

$$\frac{\partial Q}{\partial \mu} = \frac{n}{b} - \frac{(a-1)}{b}\sum_{i=1}^{n}\left(\frac{e^{\left(-\frac{x_i-\mu}{b}\right)}}{1 - e^{\left(-\frac{x_i-\mu}{b}\right)}}\right) \tag{22}$$

Equating the partial derivatives in (20), (21) and (22) to zero and replacing the parameters by their estimates gives the following equations

$$\frac{n}{\hat{a}} + \sum_{i=1}^{n} \ln\left(1 - e^{\left(-\frac{x_i-\mu}{b}\right)}\right) = 0 \tag{23}$$

$$-\frac{n}{\hat{b}} - \frac{1}{\hat{b}^2}\sum_{i=1}^{n} x_i - \frac{n\hat{\mu}}{\hat{b}^2} - \frac{(\hat{a}-1)}{\hat{b}^2}\sum_{i=1}^{n}\left(\frac{(x_i - \hat{\mu})e^{\left(-\frac{x_i-\hat{\mu}}{\hat{b}}\right)}}{1 - e^{\left(-\frac{x_i-\mu}{\hat{b}}\right)}}\right) = 0 \tag{24}$$

$$\frac{n}{\hat{b}} - \frac{(\hat{a}-1)}{\hat{b}} \sum_{i=1}^{n} \left(\frac{e^{\left(-\frac{x_i-\hat{\mu}}{\hat{b}}\right)}}{1-e^{\left(-\frac{x_i-\hat{\mu}}{\hat{b}}\right)}} \right) = 0 \tag{25}$$

Then solving (23), (24) and (25) to obtain the estimates of a, b and μ respectively.

$$\hat{a} = \frac{-n}{\sum_{i=1}^{n} \ln\left(1-e^{\left(-\frac{x_i-\mu}{b}\right)}\right)} \tag{26}$$

$$\hat{b} = -\frac{1}{n}\left(\sum_{i=1}^{n} x_i + n\hat{\mu} + (\hat{a}-1) \sum_{i=1}^{n} \left(\frac{(x_i-\hat{\mu}) e^{\left(-\frac{x_i-\hat{\mu}}{\hat{b}}\right)}}{1-e^{\left(-\frac{x_i-\mu}{\hat{b}}\right)}} \right) \right) \tag{27}$$

$$\hat{\mu} = min(x_i) - k \tag{28}$$

These results of these parameters in (26), (27) and (28) are not in closed form and can only be estimated using iterative or numerical methods. It is easy when using life data to estimate μ. It is the minimum value in the dataset less k. Where k is a very small positive value. Example k = 0.05.

4.0 Data Analysis

In this section, the reduced beta skewed Laplace distribution is applied to a real-life dataset and the results are compared with other distributions, specifically, three-parameter Weibull distribution and Weibull-Rayleigh distribution.

4.1 Photocopier Cleaning Web Failure Times Data

A data set on the failure times data from Murthy el al (2004, p. 291) concerning the failure of a photocopier cleaning web. The data is skewed to the right and mesokurtic (skewness = 0.694 and kurtosis = 2.257).

The results of the parameter estimates of three-parameter Weibull distribution and Weibull-Rayleigh (W-R) distribution are presented in Table 2. The Reduced beta skewed Laplace distribution is fitted to the data using MLE also and the result is also reported in Table 2.

The fit from the Reduced beta skewed Laplace distribution compares favorably with these other distributions in Table 2 when we compared their Bayesian Information Criteria (BIC) and their Akaike Information Criteria (AIC). Thus, all the three distributions in Table 2 provided adequate fits to the data set but Reduced beta skewed Laplace distribution is the best for this dataset because it has the least values of BIC, AIC and HQIC. This data can be classified as a small sample because there are only 14 data points in the data.

Table 2: Parameter Estimates for Failure Times Data

Distribution	Reduced beta Skewed Laplace	3-Weibull	W-R
Parameter estimates	$\hat{a}$ = 0.7968	$\hat{a}$ = 4.2973	$\hat{a}$ = 0.1820
	$\hat{b}$ = 79.1741	$\hat{b}$ = 0.4852	$\hat{k}$ = 157.5817
	$\hat{\mu}$= 25.9984	$\hat{\mu}$ = 0.9956	
Log-likelihood	-75.897	-80.2231	-81.2441
AIC	157.7942	166.4462	166.4882
BIC	159.7114	168.3634	167.7663
HQIC	157.6167	166.2687	164.3699
Skewness = 0.694, Kurtosis = 2.257, n = 14			

5.0 Conclusions

In this paper, a generalization of the Reduced beta skewed Laplace distribution is defined and studied. The Reduced beta skewed Laplace distribution is a positively skewed distribution with gaps in most cases to the right. The distribution is mostly unimodal. The shape looks like Weibull distribution.

When $a = 1$ and $\mu = 0$, Reduced beta skewed Laplace distributions becomes exponential distribution. When $a = 1$ and μ takes on any real value, Reduced beta skewed Laplace distributions becomes Laplace distribution, When $b = 1/\lambda$ and $x-\mu = z$, Reduced beta skewed Laplace distribution becomes Exponentiated Weibull distribution with $\gamma = 1$.

Reduced beta skewed Laplace distribution is unimodal since the transformer distribution, that is the skew Laplace distribution is nonnegative, continuous, and unimodal. Thus, the Reduced beta skewed Laplace distribution is unimodal since the transformed one-parameter-Kumaraswamy distribution is unimodal. The method of maximum likelihood estimation is used for estimating the parameters of Reduced beta skewed Laplace distribution. The distribution is applied to one data set and it is found to perform well in fitting the data set compared to the other two existing distributions.

We discovered that it can be used to fit other related data set e.g. amount paid to an insured in an insurance company and patient's daily arrival per hour in a hospital. The curve depicts clearly, a failure time survival and reliability curve. It is therefore recommended that Reduced beta skewed Laplace distribution be used for fitting reliability and survival data.

References

Aljarrah, M.A., Lee, C. and Famoye, F. (2014). On generating T-X family of distributions using quantile functions. *Journal of Statistical Distributions and Applications*, 1:2.

Alizadeh, M., Cordeiro, G.M., Tahir, M.H. and Mansoor, M. (2014). The Kumaraswamy Marshal-Olkin Family of Distributions. DOI: 10.1016/*j.joems*.2014.12.002.

Alzaatreh, A., Lee, C. and Famoye, F. (2013). A new method for generating families of continuous distribution. *METRON* (2013) 71:63–79

Alzaatreh, A., Lee, C. and Famoye, F. (2014). T-normal family of distributions: a new approach to generalize the normal distribution. *Journal of Statistical Distributions and Applications* 2014, 1:16

Cordeiro, GM, de-Castro, M(2011): A new family of generalized distributions. *J. Stat. Comput. Simul*. 81, 883–893

Eugene, N., Lee, C. and Famoye, F. (2012). Beta-normal distribution and its application, *Commun. Statist. Theory Meth*., 31(4), 497–512 (2002).

Famoye, F. and Govindarajulu, Z. (1998). On the Lagrange Gamma Distribution. *Computation Statistics and Data Analysis* 1998, 27, 421 – 431.

Famoye, F, Lee, C. and Eugene, N (2004). Beta-normal distribution: bimodality properties and applications. *J. Mod. Appl. Stat. Meth*. 3(1), 85–103 (2004).

Famoye, F., Lee, C., & Olumolade, O. (2005). The beta-Weibull distribution. *Journal of Statistical Theory and Applications*, 4(2), 121-136.

Kumaraswamy, P. (1980). "A generalized probability density function for double-bounded random processes". *Journal of Hydrology.* 46 (1-2): 79–88. doi:10.1016/0022-1694(80)90036-0.

Lee, C., Famoye, F. and Alzaatreh, A.Y. (2013). Methods for generating families of univariate continuous distributions in the recent decade. *Wiley Periodicals, Inc*. Vol 5, May/June 2013.

Murthy, D.N.P., Xie, M., Jiang, R. (2004). *Weibull Models.* Wiley, New York.