On The Optimum Estimator of Some Modified Ratio and Product Estimators

Umar Kabir Abdullahi*; Abubakar Yahaya

Department of Statistics,
Ahmadu Bello University,
Zaria, Nigeria.
e-mail: umarkabir9@gmail.com*

***Abstract*— In the past few decades, estimation of population mean of variable of interest using some known parameters of auxiliary variable with the aim of proposing a more efficient estimators have received wide attention from research community. In this research paper, we obtain the asymptotically optimum estimator of ratio and product estimators of Subramani and Kumarapandiyan using the Optimal C_x Approach proposed in the earlier study. The performance of the optimum estimators was compared with some optimum modified ratio and product estimators in existence using three real-life datasets considered in the literature. The proposed optimum estimators of Subramani and Kumarapandiyan are found to be uniformly better than all optimum estimators that it was compared with.**

Keywords-*Modified ratio, Modified product, Optimal C_x approach, Asymptotically optimum.*

I. INTRODUCTION

Several varieties of classes of Ratio and Product estimators have been established for the past few decades with the aim coming up with a more efficient estimate of the variable of interest by minimizing the bias and MSE using a transformation of the auxiliary variable under different sampling schemes. Among those who worked on these estimators are those proposed by [1], [2], [3], [4], [5], [6] and [7]. This study was motivated by the achievement recorded by [8] on their respective work on "On the Efficiency of Some Modified Ratio and Product Estimators- the Optimal C_x Approach"

This paper aims at improving the estimator proposed by [8] Ratio and Product estimators by using the combination of Median with some other known parameters of the auxiliary variable, such as the Coefficient of Variation, skewness, Kurtosis, Correlation, e.t.c. using simple random sampling scheme.

II. MATERIALS AND METHODS

Consider a finite population $P=(P_1,P_2,P_3,\ldots,P_N)$ of N units, a simple random sampling without replacement (SRSWOR). Let y_i and x_i, represent the value of a response variable y and auxiliary variable x, $\bar{y}$, $\bar{x}$ and $\bar{Y}$, $\bar{X}$ represent the sample and population mean of the response and auxiliary variable respectively. Furthermore, suppose in a survey dilemma, we are concerned with the estimation of population mean $\bar{y}$, when the population parameters of the auxiliary variable X are known, such as population mean $\bar{X}$,coefficient of variation C_x, coefficient of skewness $\beta_1(x)$,coefficient of Kurtosis $\beta_2(x)$ and median M_d based on sample drawn.

Shittu and Adepoju [8] proposed and obtained an optimal estimators of some modified Ratio and Product estimators by minimizing the MSE (.) from each of the estimator with respect to C_x which is obtained by taking the partial derivative of the MSE of the individual estimators with respect to C_x and equate it to zero. Then the C_x optimum is substituted back to the initial MSE to get the modified optimal estimator.

i.e $\dfrac{\partial MSE}{\partial C_x} = 0$

(1)

The usual estimator

$$T_0 = \bar{y} \tag{2}$$

with $Var(T_0) = \dfrac{1-f}{n}\bar{Y}^2 C_y^2$ (3)

The Usual Ratio and Product estimators defined by [9] and [10] are respectively given as (4) and (5) respectively

$$T_R = \bar{y}\frac{\bar{X}}{\bar{x}} \tag{4}$$

$$T_P = \bar{y}\frac{\bar{x}}{\bar{X}} \quad (5)$$

Their corresponding MSE are given by (6) and (7) respectively

$$MSE(T_R) = \frac{1-f}{n}\bar{Y}^2(C_y^2 + C_x^2 - 2\rho C_x C_y) \quad (6)$$

$$MSE(T_P) = \frac{1-f}{n}\bar{Y}^2(C_y^2 + C_x^2 + 2\rho C_x C_y) \quad (7)$$

The corresponding optimal estimator as defined by [8] are given as (8) and (9) respectively

$$MSE(T_{1(opt)}) = \frac{1-f}{n}\bar{Y}^2 C_y^2(1-\rho^2) \quad (8)$$

$$MSE(T_{2(opt)}) = \frac{1-f}{n}\bar{Y}^2 C_y^2(1+\rho^2) \quad (9)$$

The ratio and product estimator proposed by [1]

$$T_{R1} = \bar{y}\left(\frac{\bar{X} + C_x}{\bar{x} + C_x}\right) \quad (10)$$

$$T_{P1} = \bar{y}\left(\frac{\bar{x} + C_x}{\bar{X} + C_x}\right) \quad (11)$$

$$MSE(T_{R1}) = \frac{1-f}{n}\bar{Y}^2(C_y^2 + \lambda_1^2 C_x^2 - 2\lambda_1\rho C_x C_y) \quad (12)$$

$$MSE(T_{P1}) = \frac{1-f}{n}\bar{Y}^2(C_y^2 + \lambda_1^2 C_x^2 + 2\lambda_1\rho C_x C_y) \quad (13)$$

The corresponding optimal estimator define by [8] is given as

$$MSE(T_{R1[opt]}) = \frac{1-f}{n}\bar{Y}^2 C_{y1}^2(1-\rho^2) \quad (14)$$

$$MSE(T_{P1[opt]}) = \frac{1-f}{n}\bar{Y}^2 C_{y1}^2(1-\rho^2) \quad (15)$$

Where $C_{y1} = \frac{C_x\lambda_1}{\rho}$ and $\lambda_1 = \frac{\bar{X}}{\bar{X} + C_x}$, $\lambda_2 = \frac{\bar{X}}{\bar{X}+\rho}$, $\lambda_3 = \frac{\bar{X}}{\bar{X}+\beta_1}$, $\lambda_4 = \frac{\bar{X}}{\bar{X}+\beta_2}$, $\lambda_5 = \frac{\bar{X}\beta_2}{\bar{X}\beta_2 + C_x}$, $\lambda_6 = \frac{\bar{X}}{\bar{X}+M_d}$, $\lambda_7 = \frac{\bar{X}C_x}{\bar{X}C_x + M_d}$, $\lambda_8 = \frac{\bar{X}\rho}{\bar{X}\rho + M_d}$, $\lambda_9 = \frac{\bar{X}\beta_1}{\bar{X}\beta_1 + M_d}$, $\lambda_{10} = \frac{\bar{X}\beta_2}{\bar{X}\beta_2 + M_d}$.

I. ANALYSIS

Table 1: Existing Modified Ratio Estimators with their MSE and Optimum MSE and their Constants.

Estimat-ors	**MSE**	**Optimum MSE**	**Optimum C_{yi}**
$T_{R1} = \bar{y}\left(\frac{\bar{X}+C_x}{\bar{x}+C_x}\right)$ By [1]	$\frac{1-f}{n}\bar{Y}^2(C_y^2+\lambda_1^2C_x^2-2\lambda_1\rho C_xC_y)$	$\frac{1-f}{n}\bar{Y}^2C_{y1}^2(1-\rho^2)$	$C_{y1} = \frac{\lambda_1 C_x}{\rho}$
$T_{R2} = \bar{y}\left(\frac{\bar{X}+\rho}{\bar{x}+\rho}\right)$ By [11]	$\frac{1-f}{n}\bar{Y}^2(C_y^2+\lambda_2^2C_x^2-2\lambda_2\rho C_xC_y)$	$\frac{1-f}{n}\bar{Y}^2C_{y2}^2(1-\rho^2)$	$C_{y2} = \frac{\lambda_2 C_x}{\rho}$
$T_{R3} = \bar{y}\left(\frac{\bar{X}C_x+\beta_1}{\bar{x}C_x+\beta_1}\right)$ By [12]	$\frac{1-f}{n}\bar{Y}^2(C_y^2+\lambda_3^2C_x^2-2\lambda_3\rho C_xC_y)$	$\frac{1-f}{n}\bar{Y}^2C_{y3}^2(1-\rho^2)$	$C_{y3} = \frac{\lambda_3 C_x}{\rho}$
$T_{R4} = \bar{y}\left(\frac{\bar{X}C_x+\beta_2}{\bar{x}C_x+\beta_2}\right)$ By [13]	$\frac{1-f}{n}\bar{Y}^2(C_y^2+\lambda_4^2C_x^2-2\lambda_4\rho C_xC_y)$	$\frac{1-f}{n}\bar{Y}^2C_{y4}^2(1-\rho^2)$	$C_{y4} = \frac{\lambda_4 C_x}{\rho}$
$T_{R5} = \bar{y}\left(\frac{\bar{X}\beta_2+C_x}{\bar{x}\beta_2+C_x}\right)$ By [13]	$\frac{1-f}{n}\bar{Y}^2(C_y^2+\lambda_5^2C_x^2-2\lambda_5\rho C_xC_y)$	$\frac{1-f}{n}\bar{Y}^2C_{y5}^2(1-\rho^2)$	$C_{y5} = \frac{\lambda_5 C_x}{\rho}$
$T_{R6} = \bar{y}\left(\frac{\bar{X}+M_d}{\bar{x}+M_d}\right)$ By [4]	$\frac{1-f}{n}\bar{Y}^2(C_y^2+\lambda_6^2C_x^2-2\lambda_6\rho C_xC_y)$	$\frac{1-f}{n}\bar{Y}^2C_{y6}^2(1-\rho^2)$	$C_{y6} = \frac{\lambda_6 C_x}{\rho}$
$T_{R7} = \bar{y}\left(\frac{\bar{X}C_x+M_d}{\bar{x}C_x+M_d}\right)$ By [5]	$\frac{1-f}{n}\bar{Y}^2(C_y^2+\lambda_7^2C_x^2-2\lambda_7\rho C_xC_y)$	$\frac{1-f}{n}\bar{Y}^2C_{y7}^2(1-\rho^2)$	$C_{y7} = \frac{\lambda_7 C_x}{\rho}$
$T_{R8} = \bar{y}\left(\frac{\bar{X}\rho+M_d}{\bar{x}\rho+M_d}\right)$ By [6]	$\frac{1-f}{n}\bar{Y}^2(C_y^2+\lambda_8^2C_x^2-2\lambda_7\rho C_xC_y)$	$\frac{1-f}{n}\bar{Y}^2C_{y8}^2(1-\rho^2)$	$C_{y8} = \frac{\lambda_8 C_x}{\rho}$
$T_{R9} = \bar{y}\left(\frac{\bar{X}\beta_1+M_d}{\bar{x}\beta_1+M_d}\right)$ By [7]	$\frac{1-f}{n}\bar{Y}^2(C_y^2+\lambda_9^2C_x^2-2\lambda_7\rho C_xC_y)$	$\frac{1-f}{n}\bar{Y}^2C_{y9}^2(1-\rho^2)$	$C_{y9} = \frac{\lambda_9 C_x}{\rho}$
$T_{R10} = \bar{y}\left(\frac{\bar{X}\beta_2+M_d}{\bar{x}\beta_2+M_d}\right)$ By [7]	$\frac{1-f}{n}\bar{Y}^2(C_y^2+\lambda_{10}^2C_x^2-2\lambda_7\rho C_xC_y)$	$\frac{1-f}{n}\bar{Y}^2C_{y10}^2(1-\rho^2)$	$C_{y10} = \frac{\lambda_{10} C_x}{\rho}$

Table 2: Existing Modified Product Estimators with their MSE and Optimum MSE and their Constants.

Estimat-ors	**MSE**	**Optimum MSE**	**Optimum C_{yi}**
$T_{P1} = \bar{y}\left(\frac{\bar{x}+C_x}{\bar{X}+C_x}\right)$ By [1]	$\frac{1-f}{n}\bar{Y}^2(C_y^2+\lambda_1^2C_x^2+2\lambda_1\rho C_xC_y)$	$\frac{1-f}{n}\bar{Y}^2C_{y1}^2(1-\rho^2)$	$C_{y1} = \frac{\lambda_1 C_x}{\rho}$
$T_{P2} = \bar{y}\left(\frac{\bar{x}+\rho}{\bar{X}+\rho}\right)$ By [11]	$\frac{1-f}{n}\bar{Y}^2(C_y^2+\lambda_2^2C_x^2+2\lambda_2\rho C_xC_y)$	$\frac{1-f}{n}\bar{Y}^2C_{y2}^2(1-\rho^2)$	$C_{y2} = \frac{\lambda_2 C_x}{\rho}$
$T_{P3} = \bar{y}\left(\frac{\bar{x}C_x+\beta_1}{\bar{X}C_x+\beta_1}\right)$ By [12]	$\frac{1-f}{n}\bar{Y}^2(C_y^2+\lambda_3^2C_x^2+2\lambda_3\rho C_xC_y)$	$\frac{1-f}{n}\bar{Y}^2C_{y3}^2(1-\rho^2)$	$C_{y3} = \frac{\lambda_3 C_x}{\rho}$
$T_{P4} = \bar{y}\left(\frac{\bar{x}C_x+\beta_2}{\bar{X}C_x+\beta_2}\right)$ By [13]	$\frac{1-f}{n}\bar{Y}^2(C_y^2+\lambda_4^2C_x^2+2\lambda_4\rho C_xC_y)$	$\frac{1-f}{n}\bar{Y}^2C_{y4}^2(1-\rho^2)$	$C_{y4} = \frac{\lambda_4 C_x}{\rho}$
$T_{P5} = \bar{y}\left(\frac{\bar{x}\beta_2+C_x}{\bar{X}\beta_2+C_x}\right)$ By [13]	$\frac{1-f}{n}\bar{Y}^2(C_y^2+\lambda_5^2C_x^2+2\lambda_5\rho C_xC_y)$	$\frac{1-f}{n}\bar{Y}^2C_{y5}^2(1-\rho^2)$	$C_{y5} = \frac{\lambda_5 C_x}{\rho}$
$T_{P6} = \bar{y}\left(\frac{\bar{x}+M_d}{\bar{X}+M_d}\right)$ By [4]	$\frac{1-f}{n}\bar{Y}^2(C_y^2+\lambda_6^2C_x^2+2\lambda_6\rho C_xC_y)$	$\frac{1-f}{n}\bar{Y}^2C_{y6}^2(1-\rho^2)$	$C_{y6} = \frac{\lambda_6 C_x}{\rho}$
$T_{P7} = \bar{y}\left(\frac{\bar{x}C_x+M_d}{\bar{X}C_x+M_d}\right)$ By [5]	$\frac{1-f}{n}\bar{Y}^2(C_y^2+\lambda_7^2C_x^2+2\lambda_7\rho C_xC_y)$	$\frac{1-f}{n}\bar{Y}^2C_{y7}^2(1-\rho^2)$	$C_{y7} = \frac{\lambda_7 C_x}{\rho}$

$T_{P8}=\bar{y}\left(\frac{\bar{x}\rho+M_d}{\bar{X}\rho+M_d}\right)$ By [6]	$\frac{1-f}{n}\bar{Y}^2(C_y^2+\lambda_8^2C_x^2+2\lambda_7\rho C_xC_y)$	$\frac{1-f}{n}\bar{Y}^2C_{y8}^2(1-\rho^2)$	$C_{y8}=\frac{\lambda_8 C_x}{\rho}$
$T_{P9}=\bar{y}\left(\frac{\bar{x}\beta_1+M_d}{\bar{X}\beta_1+M_d}\right)$ By [7]	$\frac{1-f}{n}\bar{Y}^2(C_y^2+\lambda_9^2C_x^2+2\lambda_7\rho C_xC_y)$	$\frac{1-f}{n}\bar{Y}^2C_{y9}^2(1-\rho^2)$	$C_{y9}=\frac{\lambda_9 C_x}{\rho}$
$T_{P10}=\bar{y}\left(\frac{\bar{x}\beta_2+M_d}{\bar{X}\beta_2+M_d}\right)$ By [7]	$\frac{1-f}{n}\bar{Y}^2(C_y^2+\lambda_{10}^2C_x^2+2\lambda_{10}\rho C_xC_y)$	$\frac{1-f}{n}\bar{Y}^2C_{y10}^2(1-\rho^2)$	$C_{y10}=\frac{\lambda_{10} C_x}{\rho}$

Table 3: Statistics of the Dataset (from [14])

Parameters	Population 1	Population 2	Population 3
N	80	40	80
n	20	20	20
$\bar{Y}$	51.8264	5141.5363	51.8264
$\bar{X}$	2.8513	1221.6463	11.2646
ρ	0.9150	0.9244	0.941
C_y	0.3542	0.0557	0.3542
C_x	0.9484	0.0839	0.7507
β_1	0.6978	0.3761	0.5113
β_2	1.3005	-1.5154	2.8664
M_d	1.4800	1184.225	7.575

I. RESULTS

Table 4: The MSE and Optimum MSE of the Existing Modified Ratio Estimators and their Constants. (Population 1 and 2)

Estimators	MSE		Optimum MSE	
	Pop. 1	Pop. 2	Pop. 1	Pop. 2
$T_{R1}=\bar{y}\left(\frac{\bar{X}+C_x}{\bar{x}+C_x}\right)$ By [1]	17.19	992.29	9.91853	791.93
$T_{R2}=\bar{y}\left(\frac{\bar{X}+\rho}{\bar{x}+\rho}\right)$ By [11]	17.68	989.82	10.0964	790.843
$T_{R3}=\bar{y}\left(\frac{\bar{X}C_x+\beta_1}{\bar{x}C_x+\beta_1}\right)$ By [12]	20.66	982.41	11.2979	786.1896
$T_{R4}=\bar{y}\left(\frac{\bar{X}C_x+\beta_2}{\bar{x}C_x+\beta_2}\right)$ By [13]	12.14	995.69	8.0326	815.979
$T_{R5}=\bar{y}\left(\frac{\bar{X}\beta_2+C_x}{\bar{x}\beta_2+C_x}\right)$ By [13]	20.7801	995.69	11.1690	792.0398
$T_{R6}=\bar{y}\left(\frac{\bar{X}+M_d}{\bar{x}+M_d}\right)$ By [4]	11.1366	350.49	7.6333	204.2179
$T_{R7}=\bar{y}\left(\frac{\bar{X}C_x+M_d}{\bar{x}C_x+M_d}\right)$ By [5]	10.4605	1625.074	7.3575	5.02558
$T_{R8}=\bar{y}\left(\frac{\bar{X}\rho+M_d}{\bar{x}\rho+M_d}\right)$ By [6]	7.2256	370.15	7.17086	188.6968
$T_{R9}=\bar{y}\left(\frac{\bar{X}\beta_1+M_d}{\bar{x}\beta_1+M_d}\right)$ By [7]	6.9213	763.35	5.7922	61.8876
$T_{R10}=\bar{y}\left(\frac{\bar{X}\beta_2+M_d}{\bar{x}\beta_2+M_d}\right)$ By [7]	14.662	26797.2	8.998	6100.685

Table 5: The MSE and Optimum MSE of the Existing Modified Product Estimators and their Constants. (Population 3)

Estimators	MSE	Optimum MSE
$T_{P1}=\bar{y}\left(\frac{\bar{x}+C_x}{\bar{X}+C_x}\right)$ By [1]	109.78347	6.452479
$T_{P2}=\bar{y}\left(\frac{\bar{x}+\rho}{\bar{X}+\rho}\right)$ By [11]	107.5031	6.2528456
$T_{P3}=\bar{y}\left(\frac{\bar{x}C_x+\beta_1}{\bar{X}C_x+\beta_1}\right)$ By [12]	110.6438	6.527893
$T_{P4}=\bar{y}\left(\frac{\bar{x}C_x+\beta_2}{\bar{X}C_x+\beta_2}\right)$ By [13]	81.94399	4.094738
$T_{P5}=\bar{y}\left(\frac{\bar{x}\beta_2+C_x}{\bar{X}\beta_2+C_x}\right)$ By [13]	116.11055	7.01135
$T_{P6}=\bar{y}\left(\frac{\bar{x}+M_d}{\bar{X}+M_d}\right)$ By [4]	63.069711	2.04233
$T_{P7}=\bar{y}\left(\frac{\bar{x}C_x+M_d}{\bar{X}C_x+M_d}\right)$ By [5]	55.020140	2.4970485

$T_{P8} = \bar{y}\left(\frac{\bar{x}\rho + M_d}{\bar{X}\rho + M_d}\right)$ By [6]	61.342855	2.49705
$T_{P9} = \bar{y}\left(\frac{\bar{x}\beta_1 + M_d}{\bar{X}\beta_1 + M_d}\right)$ By [7]	44.99942	2.49705
$T_{P9} = \bar{y}\left(\frac{\bar{x}\beta_2 + M_d}{\bar{X}\beta_2 + M_d}\right)$ By [7]	90.7051866	4.81627

V. Discussion

The main focus of this research paper was to obtain the optimum estimator of Subramani and Kumarapandiyan Ratio and Product estimators which utilize the combination of median with some other known parameters of auxiliary variable in the estimation the population mean of variable of interest based on simple random sampling, which assumed the population to be a homogeneous.

In addition, we support these theoretical results numerically using real-life datasets considered in the literature as shown in Table 3.

Tables 1 & 2 provides the Mean squared error and Optimum mean squared error of some modified ratio and product estimators in existence respectively.

Tables 4 & 5 provides the numerical Mean squared error and Optimum mean squared error of some modified ratio and product estimators in existence respectively.

VI. Conclussions

Hence we conclude that the estimators in [4], [5], [6] and [7] that utilizes the combination of median and any other known parameter of the auxiliary variable (i.e. mode, coefficient of variation, kurtosis, skewness, e.t.c) have the minimum MSE, and are uniformly more efficient than those that didn't utilizes the median in simple random sampling. Thus, it is preferred to be use in practice.

References

[1] Sisodia B.V.S. and Dwivedi, V.K. (1981). A modified ratio estimator using coefficient of variation of auxiliary variable. *Journal of Indian Society of Agricultural Statistics.* 33(1):13-18.

[2] Pandey, B.N. and Durbey, Vas (1988): Modified product estimator using coefficient of variation of auxiliary variate, Assam Statistical Rev., 2(2), 64 – 66.

[3] Adewara, A. A., Rajesh, S. and Mukesh K. (2012). Efficiency of some Modified Ratio and Product Estimators using Known Values of some Population Parameters. *International Journal of Applied Sciences and Technology.* 2(2): 76-79.

[4] Subramani, J. and Kumarapandiyan, G. (2012). Modified Ratio Estimator for Population Mean Using Known median of the Auxiliary Variable. *American Journal of Mathematics and Statistics,* 3(2): 84-90.

[5] Subramani, J. and Kumarapandiyan, G. (2012). Estimation of Population Mean Using Co-efficient of Variation and median of The Auxiliary Variable. *International Journal of Probability Statistics,* 1(4): 111-118, DOI 10.5923/j.ijps.20120104.04.

[6] Subramani, J. and Kumarapandiyan, G. (2014). Estimation of Population Mean Using Known Correlation Co-efficient and median. *Journal of Probability Statistical Theory and Applications,* 13(4): 333-343.

[7] Subramani, J. and Kumarapandiyan, G. (2012). Modified Ratio Estimator Using known median and Co-efficient of Kurtosis. *American Journal of Mathematics and Statistics,* 2(4): 95-100.

[8] Shittu, O. I. and Adepoju, K. A. (2013). On the efficiency of some modified ratio and product estimators- the optimal C_x approach. *American Journal of Mathematics and Statistics.*30(5):296-299. DOI: 10.5923/j.ajms.20130305.07.

[9] Cochran, W.G. (1940). The estimation of the yield of the cereal experiments by sampling for ratio of grain to total produce. *The journal of Agricultural Science*, 30: 262-275.

[10] Murthy, M.N. (1964). Product method of estimation, *Sankhya A*, 26:69-74.

[11] Singh, H.P. and Tailor, R. (2003): Use of known correlation co-efficient in estimating the finite population means, Statistics in Transition 6 (4), 555-560.

[12] Yan, Z. and Tian, B. (2010): Ratio method to the mean estimation using co-efficient of skewness of auxiliary variable, ICICA , Part II, CCIS 106, pp. 103–110

[13] Upadhyaya, L.N. and Singh, H.P. (1999). Use of transformed auxiliary variable in estimating the finite population mean. *Biometrical Journal, 41(5): 627-636.*

[14] Singh, D. and Chaudary, F.S. (1986). *Theory and analysis of sample survey Designs.* New Age International Publisher. pp. 177-178.

[15] Okafor, F. C. (2002). *Sample Survey Theory and Applications (1st edition).* Nsukka, Nigeria.